MATH 408 and 708 Assignment #4

Due: Saturday, November 14th (11:59 p.m.)

Reading

From the textbook: Chapter 7 through Section 7.2.1. Sections 2.7 and 7.4, briefly skim 7.5.

Problems for Math 408 and Math 708

Solutions to be submitted to `Crowdmark`.

1. Consider the personal knapsack problem you made in the first homework assignment.

a. Let S be the set of 0-1 points feasible for your knapsack problem. What dimension is the face of $\text{conv}(S)$ defined by your personal knapsack inequality? If the right hand side of your personal knapsack inequality is fractional, consider also the inequality obtained by rounding down the fractional part on the right. This will also be a face of $\text{conv}(S)$. What is its dimension?

b. Use your personal knapsack inequality to derive 3 minimal cover inequalities for $\text{conv}(S)$. Note a cover is *minimal* is if removing any element from it makes it feasible. (If the LP relaxation to your personal knapsack problem gives an integer solution, work with the amended problem as in previous assignments.)

c. Solve the tightened problem given by adding the minimal cover inequalities to your basic knapsack inequality or its rounded version (from part a.) if the right hand side is fractional.

2. CCZ Exercise 3.7.

3. Consider the polytope in $\mathbb{R}^4$ generated by taking the convex hull of the points $(\pm 1, 0, 0, 0)$, $(0, \pm 1, 0, 0)$, $(0, 0, \pm 1, 0)$, and $(0, 0, 0, \pm 1)$. Describe all of its faces. How many are there in total?

4. Find four extreme points of the convex hull of the set $\{x \leq \sqrt{2}y, \quad x \geq 1, \quad x, y \in \mathbb{Z}_+\}$.

5. Recall the *stable set problem* from Homework 2. An *odd hole* in G is a cycle with an odd number of vertices and no additional edges between vertices of the cycle. If H is the an odd cycle, use the edge (2-clique) inequalities in your formulation and rounding to show that the following *odd hole* inequalities are valid for stable sets:

$$\sum_{j \in H} x_j \leq \frac{|H| - 1}{2}$$

Additional Problems for Math 708

6. Compute the Chvátal closure of $P := \{(x, y) \in [0, 1]^2 \mid 2x + 2y \leq 3\}$.

7. CCZ Exercise 3.10.

8. A system of linear inequalities $\{A\mathbf{x} \leq \mathbf{b}\}$ is *totally dual integral (TDI)* if for all $\mathbf{c} \in \mathbb{Z}^n$ such that $\{\max \mathbf{c}^t\mathbf{x} \mid A\mathbf{x} \leq \mathbf{b}\}$ has a finite optimum value, the dual linear program $\{\min \mathbf{b}^t\mathbf{y} \mid A^t\mathbf{y} = \mathbf{c},\ \mathbf{y} \geq \mathbf{0}\}$ has an integer optimum.

Show that the system $\{(x, y) \in \mathbb{R}^2 \mid x + y \leq 0, x - y \leq 0\}$ is not TDI, but that if we add the redundant inequality $x \leq 0$, the system becomes TDI.

Comment: If A is TUM, then $\{A\mathbf{x} \leq \mathbf{b}\}$ is TDI for *any* $\mathbf{b} \in \mathbb{Z}^n$.

Graduate projects and presentations

We will have graduate student presentations on Monday, December 7th and possibly on Wednesday, December 2nd.

Presentations will discuss a research paper or a brief introductory lecture on an additional topic in integer programming. This should contain substantial mathematical content and be understandable to the undergraduate students. The talks will be 20 minutes, followed by a 5 minute question period. Overheads will be submitted as part of the grading.