Due: Thursday, October 1st 4:30 p.m.

Problems for Math 408 and Math 708

1. You will have each solved a *personal knapsack problem* in your first assignment. You will now form a *team multi-constrained knapsack problem* by taking constraints for *each* student. That is: number yourselves 1,2,3 and 4. (If there are only 3 of you, use a phantom fourth student with id number 271828182.) Take each of your nine digit student id numbers[1] and add 10 to each digit to get four sequences of nine numbers $a_{11}, a_{12}, \ldots, a_{19}$; $a_{21}, a_{22}, \ldots, a_{29}$; $a_{31}, a_{32}, \ldots, a_{39}$; and $a_{41}, a_{42}, \ldots, a_{49}$. Similarly, add 10 to each of the first nine digits of π to get a sequence $b_1, b_2, \ldots, b_n$. Your *team multi-constrained knapsack problem* is:

$$\text{Maximize } \sum_{j=1}^{9} b_j x_j$$

$$\text{Subject to } \sum_{j=1}^{9} a_{ij} x_j \leq \frac{1}{2} \sum_{j=1}^{9} a_{ij} \text{ for } i = 1, 2, 3, 4$$

$$x_j \in \{0, 1\} \text{ for } j = 1, \ldots, 9.$$

a. Check if any of the solutions to your personal knapsack problems are feasible for your team multi-constrained knapsack problem.

b. If one of them is feasible (whether this happened or not), what does this imply about the team multi-constrained knapsack problem? Why?

c. Solve this team multi-constrained knapsack problem using `AMPL`. Use the `Cplex` solver. Please include a screen shot of the final solution with your written solutions.

d. How does the objective value that you found for the team multi-constrained knapsack problem relate to the objective values of your personal knapsack problems?

2. Now consider the linear programming relaxations of your personal knapsack problems and your team multi-constrained knapsack problem.

a. Check if any of the solutions to your relaxed personal knapsack problems are feasible for your relaxed team multi-constrained knapsack problem.

b. If one of them is feasible (whether this happened or not), what does this imply about the relaxed team multi-constrained knapsack problem? Why?

c. Solve this relaxed team multi-constrained knapsack problem using `AMPL`. Use the `Cplex` solver. Please include a screen shot of the final solution with your written solutions.

d. How does the objective value of the relaxed team multi-constrained knapsack problem relate to the objective values of your relaxed personal knapsack problems?

[1] If you do not want to share your student id number, take another 9 digit number for this exercise. In this case, you will need to resolve questions 1a. and 1b. from your first homework using this number.

3. The *team travelling salesperson problem* will be a geographical routing problems. Teams will pick a theme which involves at least 20 (for Math 708 groups, 50) distinct locations, with the points fairly well dispersed (not, say, nearly in a line, as often happens with Canadian-themed problems). For this assignment, your task will simply be to pick a theme and gather data about it.

Note that while this course focuses on techniques for solving discrete optimization problems, much of the work in real applications involves gathering, vetting and preparing data.

A simple way to do this is to pick a sports league such as the NHL, but more imaginative themes are encouraged. Themes are first come, first served – please consult with the instructor in making your choice. You will find the street addresses of each location, and driving distances between each pair of locations. Make a table of the relevant locations, and a table of the pairwise driving distances between them.

A five-location example is the B.C. Intercollegiate Hockey League. The teams are SFU in Burnaby, Vancouver Island University (VIU) in Nanaimo, the University of Victoria (UVic) in Victoria, UBC Okanagan (UBCO) in Kelowna, and Thompson Rivers University (TRU), with the latter two teams shared among other local institutaion. The pairwise driving distances are shown below.

Here is the table of locations:

Team	**Arena**	**Address**
SFU	Bill Copeland Sports Centre	3701 Kensington Ave., Burnaby, B.C.
VIU	Nanaimo Ice Centre	750 Third St., Nanaimo, B.C.
UVic	Ian Stewart Complex	3964 Gordon Head Rd., Victoria, B.C.
UBCO	MNP Place	4105 Gordon Dr., Kelowna, B.C.
TRU	Logan Lake Recreation Centre	31 Chartrand Avenue, Logan Lake, B.C.

And here is the table of pairwise distances in kilometres:

	SFU	VIU	UVic	UBCO	TRU
SFU	0	73.2	90.8	259.5	207.0
VIU	73.2	0	90.4	332.1	270.2
UVic	90.8	90.4	0	317.7	288.8
UBCO	259.5	332.1	317.7	0	119.5
TRU	207.0	270.2	288.8	119.5	0

Carefully document how these distances are obtained, noting the application used and the settings, including key assumptions. For instance, how is mileage on ferries counted? It is better to use a routing application that allows you to minimize distance travelled rather than time. The above is from `https://simplemaplab.com`.

This assignment will be submitted directly to the instructor by e-mail. Once you have chosen your theme, your group will get a unique identifier (such as "nhl") related to your problem, which will be included in the file names of your submission. Please include a single file named `team_hw1_name.pdf` containing all your written work, along with files `team_hw1_name.dat` and `team_hw1_name.mod` corresponding to the `AMPL` calculations in question 1c. Substitute your team's unique identifier for `name` in the file names above.