Declaration Guidelines in Test Cricket

Harsha Perera, Paramjit S. Gill and Tim B. Swartz *

Abstract

This paper considers the decision problem of when to declare during the third innings of a test cricket match. There are various factors that affect the decision of the declaring team including the target score, the number of overs remaining, the relative desire to win versus draw, and the scoring characteristics of the particular match. Decision rules are developed and these are assessed against historical matches. We observe that there are discrepancies between the optimal time to declare and what takes place in practice.

Keywords: cricket, decision rules, Gibbs sampling, parameter estimation.

1 INTRODUCTION

Although various versions of the sport of cricket have been played since the 16th century, test cricket is considered by most to be the traditional form of cricket played at the highest level. Test cricket matches between two teams may take up to five days to complete where these teams are full member nations of the International Cricket Council (ICC). Currently, there are 10 full member nations of the ICC and they are Australia, Bangladesh, England, India, New Zealand, Pakistan, South Africa, Sri Lanka, West Indies and Zimbabwe.

The laws (rules) of cricket are extensive, and are maintained by the Marylebone Cricket Club (MCC); see http://www.lords.org/laws-and-spirit/laws-of-cricket/. We now provide a very basic description of test cricket relevant to the problem examined in this paper.

A test match begins with Team A batting, and typically, Team A accumulates *runs* until 10 of their *batsmen* have been *dismissed*. This concludes the first *innings*. The runs that are accumulated during batting are the consequence of batsmen facing balls bowled by *bowlers*. A group of 6 balls bowled by the same bowler is referred to as an *over*. Team B then comes to bat during the second innings and likewise accumulates runs until 10 of their batsmen have been dismissed. Assuming that there is no *follow-on*[1], Team A bats again in the third innings. During the third innings, we are interested in the case where Team A's cumulative runs exceed Team B's runs, and this establishes a *target* score for Team B to achieve in the fourth and final innings. The question that continually faces Team A during the third innings is whether they should voluntarily terminate their innings. This is known as a *declaration*, and the decision to declare during the third innings is the subject of this paper[2].

To understand the motivation behind declaration, it is necessary to understand how matches terminate. If during the fourth innings, 10 batsmen of Team B are dismissed with Team B not reaching the target, then Team A is the winner of the match. Alternatively, if at some point during the fourth innings, Team B exceeds the target, then Team B is the winner. However, a common occurrence during the fourth innings is that the five-day time limit for the match is attained prior to Team B exceeding the target or losing its 10 wickets. In this case, the match is recorded as a draw. Therefore, during the third innings, Team A is trying to assess the merits of two subtle alternatives:

- to continue batting which leads to a more imposing target and reduces the chance of losing

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[1]At the completion of the second innings of a scheduled five-day match, if the team batting first leads by at least 200 runs, then it has the option of forcing the second innings batting team to bat again in the third innings. The sequence of batting innings according to Team A, Team B, Team B, and Team A (if required) is the result of team A enforcing the follow-on.

[2]If a team has accumulated an adequate amount of runs during its first innings (Team A) or second innings (Team B), then the team may decide to declare at that time, but these less common scenarios are not the focus of this study.

while simultaneously providing less time to dismiss Team B in the fourth innings and hence increasing the probability of a draw

- to declare, which concludes the third innings, establishes a target and leaves Team A more vulnerable to losing while simultaneously providing more time to dismiss Team B in the fourth innings

Although the declaration problem is fundamental to test cricket strategy, it has surprisingly received little attention from a quantitative perspective. To our knowledge, there are only two academic papers on the problem, Scarf and Shi (2005), and an enhanced approach given by Scarf and Akhtar (2011). We refer to the two papers collectively as SETAL (Scarf et al.). Although we later make a comparison of our results with those of SETAL, we note there are substantial differences in the two approaches. In particular, SETAL fit a multinomial regression model for match outcomes with covariates selected to improve fit. SETAL then provide estimated probabilities of match outcomes given the end of the first, second and third innings for various states of matches. SETAL also analyse the follow-on decision. Alternatively, we propose a probabilistic criterion to assess whether a team should declare in the third innings. The criterion essentially addresses whether a team is better off declaring immediately or waiting for another ball to declare. Although we do not utilize covariates, we provide a range of declaration guidelines to suit various types of matches. Another difference concerns the datasets. SETAL used a dataset of 391 test matches to estimate batting characteristics. On the other hand, we use situational datasets and ball-by-ball data. For example, to study third innings batting prior to a declaration, we consider a restricted dataset of 134 test matches where declarations occurred in the third innings. The intuition (which we confirm in section 4.1) is that immediately prior to declaring in the third innings, the batting team becomes increasingly aggressive in an attempt to pile up runs without regard to dismissals. For an overview of other decision problems related to test cricket, the reader is referred to Clarke (1998).

Section 2 begins with an exploratory data analysis. We make the observation that declaring teams rarely lose. Section 3 provides a formalization of the decision problem in terms of expected match outcomes. The solution to the problem provides a yes/no answer as to whether a team should declare at any given stage of the third innings. Various parameters are introduced, and this provides flexibility in the decision making according to the circumstances of a particular match. Parameter estimation is discussed in section 4. For the casual reader, section 4 may be skipped without jeopardizing comprehension of the main results. In contrast to our estimation procedures, Scarf, Shi and Akhtar (2011) fit negative binomial distributions to the runs scored in innings and in partnerships. They also discuss strategy in the context of third innings batting. In section 5, we provide guidelines (Table 4) as to when declaration is optimal. In some cir-

cumstances, the guidelines deviate substantially from traditional practice. Table 4 is the major contribution of the paper. In section 6, we assess the proposed decision rules. We first provide a comparison of our win-draw-loss probabilities to the probabilities obtained by SETAL. We observe reasonable agreement in many of the scenarios. We also examine historical matches and investigate how the matches may have turned out had our decision rules been implemented. We discover that our decision rules are helpful. We conclude with a brief discussion in section 7.

2   EXPLORATORY DATA ANALYSIS

For this section we collected data on 134 test matches where a declaration occurred in the third innings followed by batting in the fourth innings. These matches involved the 10 ICC teams during the period March 1, 2001 through March 31, 2012. The data are presented in various ways to provide insight on the declaration problem.

In Table 1, we record the number of times that the declaring team won, lost and drew matches. We observe that there are considerable differences amongst the 10 ICC nations. For example, some of the teams (e.g. Bangladesh, West Indies and Zimbabwe) rarely declare. This may not be a reflection of their cautiousness. Rather, these are weaker teams and they are rarely in a position to declare. We also observe that Australia declares often and has a higher percentage of wins in the declared matches than the other ICC teams. This suggests that Australia is bolder with respect to declaring. Although Australia appears to more readily declare, they do so without increasing their loss percentage. Generally, it seems that declaring rarely results in a loss, and that declaring teams win at roughly the same rate at which they draw.

Team	N	Declared Matches	Wins	Losses	Draws
Australia	62	29	22 (76%)	1 (3%)	6 (21%)
Bangladesh	45	3	1 (33%)	0 (0%)	2 (67%)
England	62	24	12 (50%)	1 (4%)	11 (46%)
India	46	16	6 (38%)	0 (0%)	10 (63%)
New Zealand	38	6	4 (67%)	0 (0%)	2 (33%)
Pakistan	32	11	7 (64%)	0 (0%)	4 (36%)
South Africa	47	22	7 (32%)	1 (5%)	14 (64%)
Sri Lanka	47	16	11 (69%)	0 (0%)	5 (31%)
West Indies	35	3	1 (33%)	0 (0%)	2 (67%)
Zimbabwe	21	4	2 (50%)	0 (0%)	2 (50%)
Overall	435	134	73 (51%)	3 (2%)	67 (47%)

Table 1: Match outcomes for teams that declared in the third innings. The column N is the total number of matches over the period where the team batted in the first and third innings.

In Figure 1, we provide a scatterplot of the target set by the declaring team versus the remaining overs in the match at the time of declaration. The remaining overs were calculated by noting that test cricket allows up to 90 overs in a day. As anticipated, the target is greater when more overs remain. We again observe that declaring teams lose very few matches (triangle symbol) which suggests that teams are often cautious in the sense that they declare late in matches. The scatterplot also suggests that with more than 150 overs remaining, the declaring team usually wins.
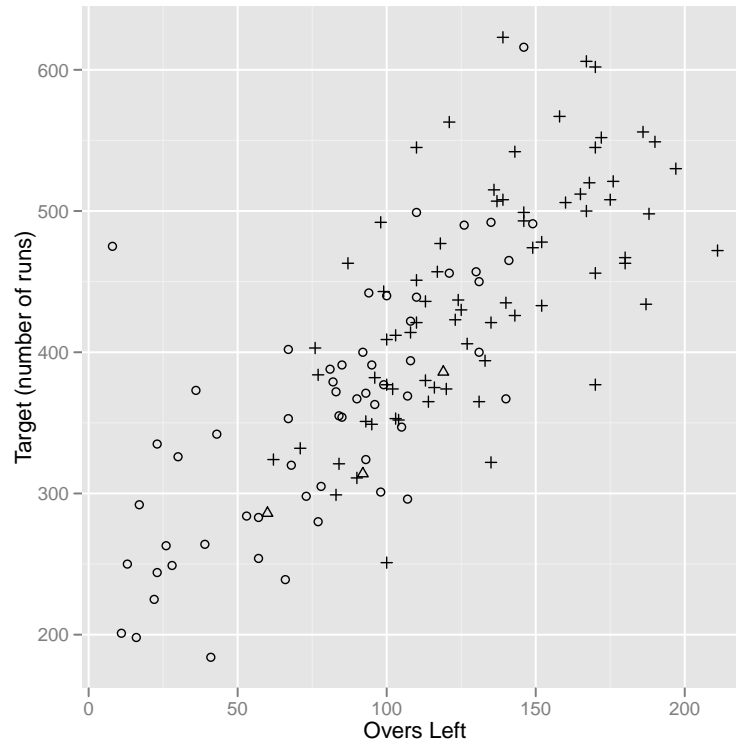

Figure 1: Scatterplot of the target versus the remaining overs for matches with declarations in the third innings. Plus/circle/triangle symbols indicate that the declaring team has gone on to win/draw/lose the match.

Figure 2 is a scatterplot of the fourth innings runs versus the overs consumed in the fourth innings. We observe that the number of runs scored is roughly proportional to the number of overs. This may be a little surprising since better batsmen tend to bat earlier in batting lineups. We also observe that there does not appear to be any systematic difference in the number of runs scored between batting teams that are all out (plus symbol) and those that are not all out (circle symbol). The variability of the number of runs scored increases with the numbers of overs consumed. All of these observations are relevant to the modelling assumptions made in sections 4.2 and 4.3.
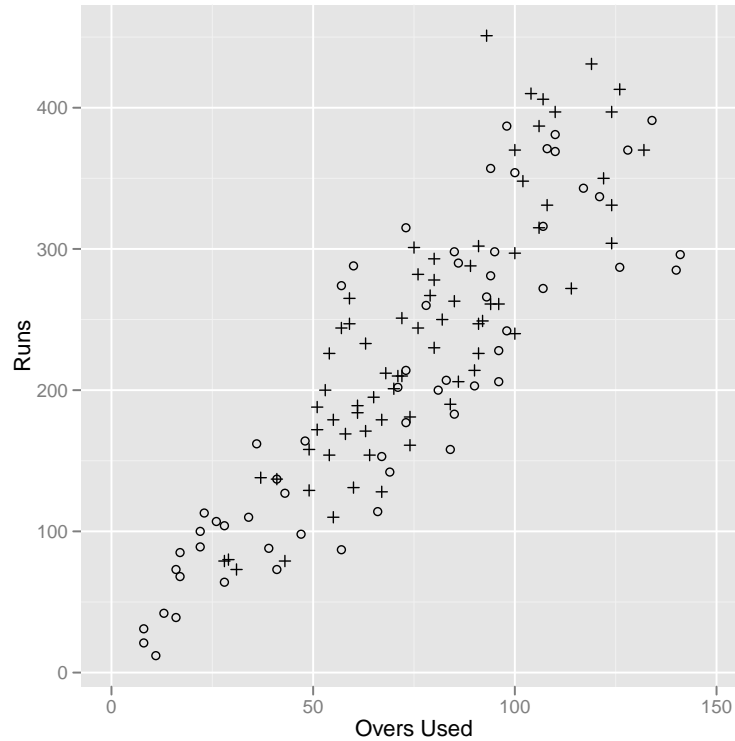
Figure 2: Scatterplot of fourth innings runs versus overs consumed for matches with declarations in the third innings. Plus (circle) symbols indicate that the batting team was all out (not all out) at the end of the fourth innings.

In Figure 3, we provide a more comprehensive scatterplot of runs scored versus overs used. The scatterplot involves all of the 1849 innings arising from test matches involving ICC nations between March 1, 2001 and March 31, 2012. Whereas this data is used for parameter estimation in section 4, the key feature at this stage is that the scatterplot is similar to the scatterplot in Figure 2. This suggests that the distribution of runs scored in the fourth innings following a third innings declaration does not differ from the distribution of runs scored in general.

3   THE DECISION PROBLEM

In formulating the decision problem, we consider Team A batting in the third innings, pondering whether they should declare. We denote a win and a draw for Team A as W and D respectively, and let y be the number of *overs* remaining in the match. Again, the value y can be obtained at any point during a match since test cricket allows 90 overs in a day. Since we are interested in how the situation changes for Team A over time, we let y' be the number of overs y remaining in the match less one ball bowled. To complete the notation, we let Q_y correspond to the decision
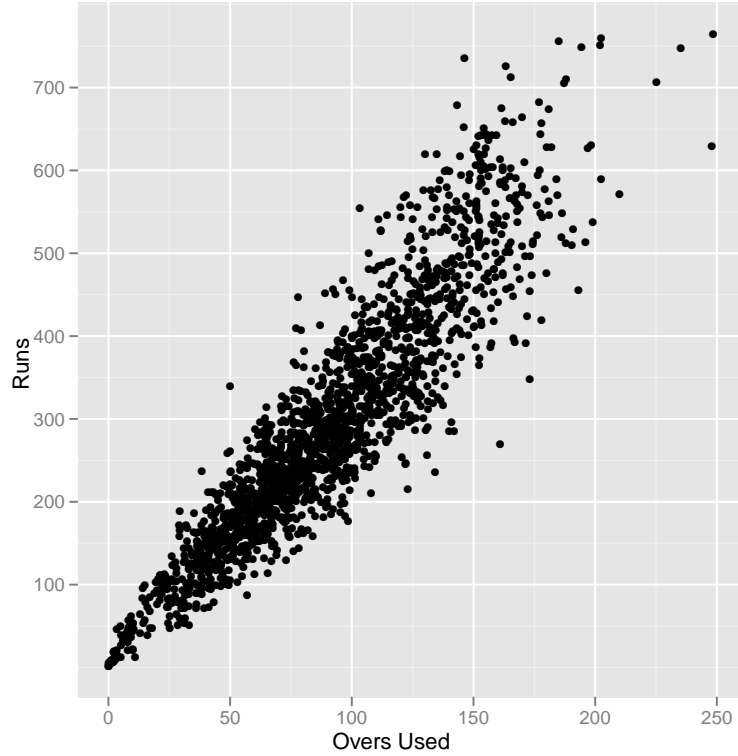
Figure 3: Scatterplot of runs scored (in an innings) versus overs used in test matches involving ICC nations between March 1, 2001 and March 31, 2012.

that Team A declares with y overs remaining and we let T_y be the corresponding target score with y overs remaining.

In most sports (soccer notably excluded), teams receive x points for winning a match, $x/2$ points for a draw and zero points for a loss (ignoring bonus points). This is an intuitive scoring system where a draw is assigned half the value of a win. Usually, $x = 2$, but in some sports and competitions, such as rugby union (e.g. the Rugby World Cup), $x = 4$. Although points are not explicitly awarded in a test cricket series, we note that the above point system (with $x = 2$) corresponds to how cricket series are decided. For example, a five-match series is considered drawn should either a 2-1-2 result or a 1-3-1 result occur according to Win-Draw-Loss. In these two cases, both results yield five points. Accordingly, we assert that Team A should declare when the bowling of an additional ball causes their expected number of match points to decrease. Using the notation introduced above, Team A should declare with y overs remaining if

$$
\begin{aligned}
& 2 \cdot \mathrm{Prob}(W \mid T_y \; ; \; Q_y) + 1 \cdot \mathrm{Prob}(D \mid T_y \; ; \; Q_y) \\
> \quad & 2 \cdot \mathrm{Prob}(W \mid T_y \; ; \; Q_{y'}) + 1 \cdot \mathrm{Prob}(D \mid T_y \; ; \; Q_{y'}) \, .
\end{aligned}
\tag{1}
$$

Now, there may be circumstances where winning is imperative. That is, for Team A, a draw

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is no better than a loss. An example of this may occur in a four-match test cricket series where after three matches, Team A has one win and two losses. For Team A, a win in the fourth and final match will result in a drawn series whereas anything less than a win in the fourth match results in a lost series. In this case, we modify (1) and assert that Team A should declare with y overs remaining if

$$\text{Prob}(W \mid T_y \ ; \ Q_y) \ > \ \text{Prob}(W \mid T_y \ ; \ Q_{y'}) \ . \tag{2}$$

It may also be possible that the objective of Team A is to avoid losing a match at all costs. For example, Team A may be leading a five match series with one win and three draws. Either a win or a draw in the fifth match will allow them to win the series. However, a loss in the fifth match will result in a drawn series. When Team A is batting in the third innings, their probability of losing the match decreases as they continue to bat. This is because they are simultaneously accumulating runs and providing less time for Team B to score runs. Therefore, if the objective is strictly not to lose, Team A should never declare. In this scenario, there is no need for a quantitative analysis.

The evaluation of (1) and (2) can be simplified by invoking the total law of probability. Accordingly, consider the possible outcomes that can occur on the next ball bowled:

$$\begin{aligned} B_{-1} &\equiv \text{dismissal} \\ B_i &\equiv i \text{ runs scored}, \quad i = 0, \ldots, 6 \end{aligned}$$

where we exclude the possibility of extra runs that can be accumulated via wide-balls and no-balls. In fact, we do account for extra runs but avoid the presentation here to maintain the simplified notation. It follows that

$$\begin{aligned} \text{Prob}(W \mid T_y \ ; \ Q_{y'}) &= \text{Prob}(WB_{-1} \cup WB_1 \cup \cdots \cup WB_6 \mid T_y \ ; \ Q_{y'}) \\ &= \sum_{i=-1}^{6} \text{Prob}(WB_i \mid T_y \ ; \ Q_{y'}) \\ &= \sum_{i=-1}^{6} p_i \, \text{Prob}(W \mid T_{y'(i)} \ ; \ Q_{y'}) \end{aligned} \tag{3}$$

where $T_{y'(i)} = T_y + i\delta(i \neq -1)$, δ is the indicator function and

$$p_i = \text{Prob}(B_i) \tag{4}$$

is the probability of B_i, $i = -1, \ldots, 6$, where Team A is batting and is on the verge of declaring. Similarly,

$$\text{Prob}(D \mid T_y \ ; \ Q_{y'}) \ = \ \sum_{i=-1}^{6} p_i \, \text{Prob}(D \mid T_{y'(i)} \ ; \ Q_{y'}) \ . \tag{5}$$

We therefore observe that the terms in (3) have the same form as $\text{Prob}(W \mid T_y \; ; \; Q_y)$ and that the terms in (5) have the same form as $\text{Prob}(D \mid T_y \; ; \; Q_y)$. This simplifies the evaluation of the inequalities (1) and (2).

At this stage, we provide a partial recap of what needs to be done in order to decide whether Team A should declare in the third innings with y overs remaining and a target of T_y. Team A must first decide whether a win is essential. If so, then the simpler inequality (2) provides the appropriate criterion; otherwise inequality (1) is the appropriate criterion. To evaluate (1) and (2), we need to first estimate the batting characteristics of Team A in the third innings given by the parameters $p_{-1}, \ldots, p_6$ in (4). Secondly, we need to estimate $\text{Prob}(W \mid T_y \; ; \; Q_y)$ which is dependent on the batting performance of Team B in the fourth innings. Thirdly, we may need to estimate $\text{Prob}(D \mid T_y \; ; \; Q_y)$ which is also dependent on the batting performance of Team B in the fourth innings. We elaborate on all three estimation procedures in section 4.

4   PARAMETER ESTIMATION

Data were collected from the Cricinfo website http://www.espncricinfo.com/ where matches were filtered using Statsguru. In cases where ball-by-ball data were required, we accessed matches from the Cricinfo Archive where Commentary logs were downloaded. The Commentary logs appear to be the only comprehensive source of ball-by-ball data. A proprietary R script was then used to parse the commentary logs and obtain ball-by-ball data in a convenient format.

4.1   Estimation of $p_{-1}, \ldots, p_6$

The parameters $p_{-1}, \ldots, p_6$ given in (4) describe the batting characteristics of Team A when they are on the verge of declaring. With ball-by-ball data, we are able to obtain the proportions corresponding to the batting events $B_{-1}, \ldots, B_6$. We collected data on 134 test matches for which a declaration occurred in the third innings. These matches involved the 10 ICC teams during the period March 1, 2001 through March 31, 2012.

In Table 2, we present the results for varying overs remaining in the third innings. We observe that teams that are on the verge of declaring (i.e. 5 and 10 overs remaining) bat more aggressively than they generally do throughout the third innings. For example, the proportion of 6's is more than three times as large in the second and third rows of Table 2 than in the first row. This is sensible since teams on the verge of declaring are trying to pile up runs without regard to dismissals. We also note that there is greater stability in the proportions $\hat{p}_1, \ldots, \hat{p}_6$ than in $\hat{p}_{-1}$ and $\hat{p}_0$ when comparing the results in the second and third rows of Table 2. This is good news since only $p_1, \ldots, p_6$ contribute to the target score $T_{y'(i)}$ in (3) and (5). For our

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analyses in section 5, we use the estimates from the third row of Table 2 to describe Team A's batting characteristics when they are on the verge of declaring in the third innings.

Overs Remaining	Balls Bowled	$\hat{p}_{-1}$	$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$	$\hat{p}_5$	$\hat{p}_6$
All	62948	0.012	0.664	0.206	0.040	0.009	0.065	0.000	0.005
10	7496	0.027	0.519	0.293	0.060	0.007	0.080	0.000	0.014
5	3659	0.038	0.490	0.306	0.063	0.008	0.079	0.000	0.016

Table 2: Sample proportions corresponding to the parameters $p_{-1}, \ldots, p_6$ in (4). The proportions are obtained for varying overs remaining in the third innings.

Now it is possible that the estimates from the third row in Table 2 vary according to the quality of Team A. We therefore repeated the above exercise and obtained team specific estimates over the last five years. We discovered that there are only minor differences in the characteristics of the 10 teams. These differences also proved inconsequential with respect to the optimal time to declare.

4.2 Estimation of $\mathrm{Prob}(W \mid T_y \; ; \; Q_y)$

Recall that $\mathrm{Prob}(W \mid T_y \; ; \; Q_y)$ is the probability that Team A wins the match given that they declare with y overs remaining in the match and the target score is T_y. The event W occurs if Team B scores fewer than T_y runs and is dismissed in y overs or less. Therefore

$$
\begin{aligned}
\mathrm{Prob}(W \mid T_y \; ; \; Q_y) \;&=\; \mathrm{Prob}(R_y < T_y \; \cap \; O_1 + \cdots + O_{10} \le y) \\
&=\; \sum_{i=0}^{y} q_i \, \mathrm{Prob}(R_y < T_y \mid O_1 + \cdots + O_{10} = i) \quad (6)
\end{aligned}
$$

where R_y is the number of runs scored by Team B in the fourth innings with y overs available, O_i is the number of overs that it takes for the ith wicket to fall and

$$
q_i = \mathrm{Prob}(O_1 + \cdots + O_{10} = i) \quad (7)
$$

for $i = 0, \ldots, y$.

The estimation of q_i in (7) is carried out by modelling the O_j's as independent geometric variables. Each O_j has a single parameter that can be estimated from the batting records in the fourth innings following a third innings declaration. When the geometric parameters are obtained, then q_i can be calculated via the convolution formula. The convolution formula is useful when the distribution of a sum of random variables is required.

There is a catch in the estimation of the geometric parameters, and this involves censored data. Consider matches where we wish to estimate the geometric parameter r_j corresponding

to O_j. The probability mass function for O_j is

$$\text{Prob}(O_j = k) = r_j(1 - r_j)^{k-1} \qquad k = 1, 2, \ldots \tag{8}$$

From the n_j fourth innings matches where there were at least $j - 1$ dismissals, we have data $O_j^{(1)}, \ldots, O_j^{(l_j)}, O_j^{(l_j+1)}, \ldots, O_j^{(n_j)}$ where the first l_j observations are not censored (i.e. these are the number of overs that it took for the jth wicket to fall) and the remaining $n_j - l_j$ observations are censored (i.e. these are the number of overs from wicket $j - 1$ until the end of the match). The likelihood corresponding to $O_j^{(1)}, \ldots, O_j^{(l_j)}, O_j^{(l_j+1)}, \ldots, O_j^{(n_j)}$ is therefore given by

$$\prod_{k=1}^{l_j} r_j(1 - r_j)^{O_j^{(k)}-1} \prod_{k=l_j+1}^{n_j} (1 - r_j)^{O_j^{(k)}} = \left(\frac{r_j}{1 - r_j}\right)^{l_j} (1 - r)^{\sum_{k=1}^{n_j} O_j^{(k)}}$$

from which we obtain the maximum likelihood estimator $\hat{r}_j = l_j / \sum_{k=1}^{n_j} O_j^{(k)}$. For our geometric model, the only assumption is that the outcomes of balls (dismissal or not) consist of independent and identically distributed Bernoulli trials.

In section 5 we present optimal declaration guidelines, and this is done in an average sense. However, we also want to take into account both weak and strong teams. For the estimation of $r_1, \ldots, r_{10}$, we define a notional weak/strong team as one whose parameter estimates lie one standard deviation above/below the maximum likelihood estimates. For these calculations, we use a Fisher information approximation to the asymptotic variance of the maximum likelihood estimator.

Table 3 provides the maximum likelihood estimates of the geometric parameters $r_1, \ldots, r_{10}$ for an average team using the fourth innings data. We also provide estimates for relatively weak and relatively strong teams. As expected, we observe that the geometric parameters are generally increasing as the number of wickets fall. This implies that batsmen further down the batting order are more likely to be dismissed. The interesting exception is the opening partnership where dismissals occur at a higher rate than some of the other early partnerships. This may be due to the fact that there is an adjustment period at the beginning of an innings where batsmen need to acquire comfort with a new ball, changed field conditions, new bowlers, etc.

Team B	$\hat{r}_1$	$\hat{r}_2$	$\hat{r}_3$	$\hat{r}_4$	$\hat{r}_5$	$\hat{r}_6$	$\hat{r}_7$	$\hat{r}_8$	$\hat{r}_9$	$\hat{r}_{10}$
Weak	0.117	0.085	0.082	0.087	0.083	0.102	0.142	0.179	0.193	0.228
Average	0.108	0.079	0.076	0.081	0.077	0.094	0.131	0.166	0.179	0.212
Strong	0.099	0.073	0.070	0.075	0.071	0.086	0.120	0.153	0.165	0.196

Table 3: Estimates of the geometric parameters r_j as defined in (8) for weak, average and strong teams.

The final estimation procedure in this subsection concerns the conditional distribution of R_y given $O_1 + \cdots + O_{10} = i$ for the probability specified in (6). We first postulate that the number of runs R_y scored by Team B in the fourth innings with y overs available given that they are all out in i overs, $i = 1, \ldots, y$ can be approximated via

$$[R_y \mid O_1 + \cdots + O_{10} = i] \sim \text{Normal}(\mu \cdot i, \sigma^2 \cdot i) . \tag{9}$$

The proposed distribution (9) only requires the estimation of the two parameters μ and σ. The rationale of the normal distribution is based on a Central Limit Theorem effect whereby runs are accumulated as the sum over many balls. By considering the runs scored and the associated number of overs used in the fourth innings data, we obtained maximum likelihood estimates $\hat{\mu} = 3.02$ and $\hat{\sigma} = 5.95$.

Whereas (9) provides an appealing rough-and-ready distribution for runs scored, we want to ensure that it is realistic. Some doubt concerning the adequacy of (9) may be seen by considering the case of $i = 50$ overs. Here, $\hat{\mu} \cdot i \pm 2\hat{\sigma}\sqrt{i}$ provides the interval $(116.6, 185.4)$ which seems too narrow for the number of runs scored when compared to Figure 2.

Alternatively, we consider a more data-driven approach with minimal assumptions on the mean-variance structure. Specifically, we propose

$$[R_y \mid O_1 + \cdots + O_{10} = i] \sim \text{Normal}(\mu_i, \sigma_i^2) \tag{10}$$

where both the means μ_i and the variances σ_i^2 are assumed to be increasing relative to the number of overs i. By introducing reference priors $[\mu_i] \propto 1$ and $[\sigma_i^2] \propto 1/\sigma_i^2$ subject to the order constraints, we have a high dimensional Bayesian model. Let $R_{i1}, \ldots, R_{in_i}$ be the number of runs scored in the n_i fourth innings that lasted exactly i overs. Then the posterior means of the μ_i and σ_i are readily obtained through a Gibbs sampling algorithm where full conditional distributions are given by

$$
\begin{aligned}
[\mu_i \mid \cdot] &\sim \text{Normal}(\bar{R}_i, \sigma_i^2/n_i) & \mu_{i-1} < \mu_i < \mu_{i+1} \\
[\sigma_i^2 \mid \cdot] &\sim \text{Inverse Gamma}(n_i/2, 2/(\textstyle\sum_{j=1}^{n_i}(R_{ij} - \mu_i)^2) & \sigma_{i-1} < \sigma_i < \sigma_{i+1}, \ n_i > 1 \\
[\sigma_i^2 \mid \cdot] &\propto 1/\sigma_i^2 & \sigma_{i-1} < \sigma_i < \sigma_{i+1}, \ n_i = 1 .
\end{aligned}
$$

The generation of variates from constrained distributions is facilitated using inversion. When a random variable X has a cumulative distribution function F and is further constrained to the interval (a, b), inversion proceeds by obtaining $x = F^{-1}(F(a) + u(F(b) - F(a)))$ where $u \sim \text{uniform}(0, 1)$.

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4.3 Estimation of $\text{Prob}(D \mid T_y \; ; \; Q_y)$

Recall that $\text{Prob}(D \mid T_y \; ; \; Q_y)$ is the probability that Team A draws the match given that they declare with y overs remaining and a target score T_y. The event occurs if Team B scores no more than T_y runs and is not dismissed in the y overs available. Therefore

$$
\begin{aligned}
\text{Prob}(D \mid T_y \; ; \; Q_y) \;\; &= \;\; \text{Prob}(R_y \leq T_y \;\cap\; O_1 + \cdots + O_{10} > y) \\
&= \;\; \text{Prob}(R_y \leq T_y \mid O_1 + \cdots + O_{10} > y) \, \text{Prob}(O_1 + \cdots + O_{10} > y) \quad (11)
\end{aligned}
$$

where R_y is the number of runs scored by Team B in the fourth innings with y overs available, O_i is the number of overs that it takes for the ith wicket to fall and

$$
\text{Prob}(O_1 + \cdots + O_{10} > y) \;\; = \;\; 1 - \sum_{i=0}^{y} q_i \tag{12}
$$

with q_i defined in (7).

The estimation of (12) is the same as that described in subsection 4.2. As for the estimation of the first term in (11), we propose the distribution

$$
[R_y \mid O_1 + \cdots + O_{10} > y] \sim \text{Normal}(\mu_y, \sigma_y^2) \; . \tag{13}
$$

When comparing (10) to (13), we argue that it is the number of overs used which is paramount in the run distribution. In fact, the scatterplot of runs versus overs used (when all-out) does not differ materially from the scatterplot of runs versus overs used (when not all-out). The synthesis of the two scatterplots appears in Figure 3.

For the estimation of the constrained parameters μ_i and σ_i, we considered the 1849 innings corresponding to matches between ICC nations from March 1, 2001 through March 31, 2012. Using the Gibbs sampling algorithm described above, examples of estimates that we obtained are $\hat{\mu}_{50} = 169.6$, $\hat{\mu}_{100} = 316.9$, $\hat{\mu}_{150} = 508.6$, $\hat{\sigma}_{50} = 35.3$, $\hat{\sigma}_{100} = 55.1$ and $\hat{\sigma}_{150} = 75.1$. These estimates appear sensible when compared to both Figure 2 and Figure 3. To gain some further intuition for the estimates, consider the probability of scoring at least 400 runs in 100 overs of batting. The probability is given by $\text{Prob}(Z \geq (400 - 316.9)/55.1) = 0.07$ where Z is a standard normal random variable.

Posterior standard deviations of the parameters are also readily available from the Gibbs sampling algorithm and they provide us with a measure of confidence in the estimates. For example, we obtained $\text{sd}(\mu_{50}) = 3.7$, $\text{sd}(\mu_{100}) = 5.5$, $\text{sd}(\mu_{150}) = 7.0$, $\text{sd}(\sigma_{50}) = 1.2$, $\text{sd}(\sigma_{100}) = 3.0$ and $\text{sd}(\sigma_{150}) = 5.8$. We observe that the posterior standard deviations increase for increasing numbers of overs. This is somewhat expected as data becomes more scarce for increasing numbers of overs. We therefore did not produce estimates beyond $i = 180$ overs. The upper value of

$i = 180$ overs corresponds to two full days of batting.

Again, we also want to account for relatively strong and weak teams. We define a relatively strong (weak) Team B as one whose expected number of runs $\hat{\mu}_i$ in i overs of batting is 20% above (below) the posterior mean. In the above example, we obtain $\hat{\mu}_{100} = 380.3$ (253.5) runs for a relatively strong (weak) Team B.

5 RESULTS

At this stage, we provide some practical declaration guidelines. First, a decision maker needs to determine how badly Team A wants to win a given match. If winning the match is of the utmost importance, then criterion (2) is used for decision making; otherwise criterion (1) is used. In the midst of a match, it may be inconvenient for a decision maker to access a computer, input parameter estimates and evaluate the relevant inequality. Instead, we have produced Table 4 which summarizes optimal declaration targets for specified overs remaining in a match. The table takes into account whether Team B is relatively weak, average or strong compared to Team A. For a particular situation, Table 4 was constructed by systematically increasing the target for a fixed number of overs until the relevant inequality was satisfied. Table 4 is the major contribution of the paper, and it is our hope that it may be utilized for better decision making in test cricket.

A stunning feature of Table 4 is that the decision to declare depends critically on how badly Team A wants to win. Under criterion (2) where Team A is desperate to win, they should declare with a much smaller target than under criterion (1). For example, with 90 overs remaining in a match, a desperate Team A (criterion (2)) should declare with a target of 296 runs against an average opponent. This is contrasted with a target of 359 runs using criterion (1) against an average opponent. The difference sensibly dissipates with a large number of overs remaining since Team B will most likely be made out and there is little chance that the match will end in a draw. For example, with 180 overs remaining in a match, a desperate Team A should declare with a target of 629 runs against an average opponent. This is contrasted with a comparable target of 652 runs using criterion (1). We also observe that the quality of Team B has an impact on the declaration guidelines where weaker/stronger opponents naturally require a lower/higher target. For example, under criterion (1) with 120 overs remaining in the match, Team A should declare with a target of 389/467/548 runs against weak/average/strong opponents. We also observe that the recommendations in Table 4 can differ substantially from common practice. For example, Figure 1 suggests that with 50 overs remaining, teams declare when the target is roughly 300 runs. In contrast, Table 4 requires far fewer runs than that, a target of 254 runs against an average team using criterion (1). With large numbers of overs remaining, the opposite

phenomenon takes place. That is, Table 4 indicates that teams should declare with a higher target than is common practice. For example, when 150 overs are remaining, common practice suggests a declaration target of roughly 500 runs whereas Table 4 provides a target of 592 runs against an average team using criterion (1). The target guidelines for large numbers of overs may first appear to be in conflict with Figure 1. In Figure 1, we observe that the declaring team (Team A) nearly always wins when there is a large number of overs remaining, and this is suggestive that they could declare earlier. However, there were only 21 matches where teams declared with more than 150 overs remaining. Although Team B was dismissed and did not reach its target in this small sample of matches, Figure 3 suggests that teams have the run scoring capability of doing so.

Overs Remaining	Criterion (1) - Win or Draw			Criterion (2) - Must Win		
	Weak	Average	Strong	Weak	Average	Strong
50	214	254	295	131	161	192
60	229	274	320	158	193	228
70	253	304	356	189	228	269
80	274	331	389	219	263	309
90	298	359	422	247	296	346
100	331	399	469	277	331	387
110	361	436	513	311	371	433
120	389	467	548	343	409	477
130	423	507	595	377	448	522
140	457	548	643	413	491	571
150	494	592	679	450	535	623
160	527	612	704	486	576	648
170	556	631	728	521	605	674
180	585	652	752	552	629	699

Table 4: Optimal target scores for declaration. The targets are calculated for a specified number of overs remaining in a match under both criterion (1) and criterion (2) and according to whether Team B is a weak, average or strong scoring team relative to Team A.

We note that the quality of Team B in Table 4 is a characterization of its ability to score runs in the fourth innings. Clearly, the capacity for Team B to score runs also depends on the bowling and fielding standard of Team A. Therefore, a decision ought to take Team A into account when referring to Table 4. For example, if Team A is a strong bowling team and Team B is an average batting team, then one may want to refer to the "Weak" category in Table 4. A decision maker may alternatively choose intermediate values between the three categories of weak, average and strong.

A practical implementation of the above discussion is to let r_A denote the average number of

runs allowed by Team A and let r_B denote the average number of runs scored by Team B. We may then take $P = (r_B - r_A)/r_A$ 100% as the percentage that Team B is better at batting than Team A. Since weak and strong teams in Table 4 are based on a sizeable 20% criterion, we may then interpolate table values based on P.

6   ASSESSING THE DECISION RULES

Associated with our decision rules in Table 4 are win, loss and draw probabilities corresponding to Team A where Team B is assumed to be a relatively average team. In Table 5, we provide these estimated probabilities for declarations in the third innings, and we compare these probabilities against the probabilities reported by SETAL. Specifically, we use the SETAL results from Table 4 of their 2011 paper which represent standard conditions.

From Table 5, we first observe that our probabilities generally correspond to intuition. For example, for a given target score, it becomes more probable for Team A to lose a match as the number of remaining overs increase. The same is true with the SETAL probabilities. We also observe general agreement between our probabilities and the SETAL probabilities which is reassuring since the two methods of estimation are dramatically different. In the case of substantial differences, there does not appear to be any obvious pattern. One of the largest differences occurs in the cell with 60 overs remaining and a target score of 200 runs. Referring to Figure 3, although it is plausible to score at least 200 runs from 60 overs, it seems that the probability of the event may be less than 0.5. Recall that the horizontal axis in Figure 3 is overs used, and frequently, Team B is dismissed prior to 60 overs. This suggests that the SETAL L (loss) probability of 0.49 may be too large. Perhaps the true probability of L lies somewhere between 0.29 and 0.49. We also comment on some of the other substantial differences between our probabilities and the SETAL probabilities. These larger differences tend to occur in the cells with 100-160 overs remaining and target scores between 250 and 350 runs. Referring to Figure 1, these cells correspond to scenarios where declarations rarely occur. In these instances, we provide larger L probabilities than SETAL. Although we do not have great intuition for these scenarios, the reason teams do not declare in these cases is because they believe that there is considerable probability that they may lose.

However, we stress that the focus of the paper is not whether our probabilities are preferable to the SETAL probabilities. The two sets of probabilities are somewhat comparable, and this is comforting. The more important issue is whether our decision rules regarding declaration lead to more favourable outcomes than what is observed in current practice.

We now evaluate our decision rules given in Table 4 via a comparison with actual data. We take a validation approach whereby we apply our results to matches that were not used in the

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		Overs Remaining					
Target		60	80	100	120	140	160
	W	0.09(0.19)	0.19(0.23)	0.20(0.25)	0.20(0.25)	0.20(0.25)	0.20(0.24)
200	D	0.62(0.33)	0.10(0.14)	0.00(0.05)	0.00(0.02)	0.00(0.01)	0.00(0.00)
	L	0.29(0.49)	0.71(0.63)	0.80(0.70)	0.80(0.73)	0.80(0.75)	0.80(0.76)
	W	0.11(0.23)	0.30(0.37)	0.39(0.45)	0.40(0.49)	0.40(0.50)	0.40(0.50)
250	D	0.87(0.57)	0.38(0.32)	0.03(0.14)	0.00(0.05)	0.00(0.02)	0.00(0.01)
	L	0.02(0.20)	0.32(0.32)	0.58(0.41)	0.60(0.46)	0.60(0.48)	0.60(0.50)
	W	0.11(0.21)	0.34(0.40)	0.54(0.58)	0.57(0.69)	0.58(0.74)	0.56(0.75)
300	D	0.89(0.73)	0.61(0.49)	0.13(0.25)	0.02(0.10)	0.00(0.04)	0.00(0.01)
	L	0.00(0.06)	0.05(0.11)	0.33(0.17)	0.41(0.21)	0.42(0.23)	0.44(0.24)
	W	0.11(0.17)	0.35(0.36)	0.60(0.59)	0.71(0.77)	0.73(0.85)	0.72(0.89)
350	D	0.89(0.82)	0.65(0.61)	0.29(0.36)	0.08(0.16)	0.01(0.06)	0.00(0.02)
	L	0.00(0.01)	0.00(0.03)	0.11(0.05)	0.21(0.07)	0.26(0.08)	0.28(0.09)
	W	0.11(0.13)	0.35(0.29)	0.61(0.53)	0.79(0.75)	0.85(0.88)	0.86(0.94)
400	D	0.89(0.87)	0.65(0.70)	0.37(0.45)	0.14(0.22)	0.03(0.09)	0.01(0.03)
	L	0.00(0.00)	0.00(0.01)	0.02(0.02)	0.07(0.02)	0.12(0.03)	0.13(0.03)
	W	0.11(0.09)	0.35(0.23)	0.62(0.46)	0.81(0.70)	0.90(0.87)	0.91(0.94)
450	D	0.89(0.91)	0.65(0.77)	0.38(0.54)	0.18(0.29)	0.06(0.13)	0.04(0.05)
	L	0.00(0.00)	0.00(0.00)	0.00(0.00)	0.01(0.01)	0.04(0.01)	0.05(0.01)

Table 5: The estimated win (W), draw (D) and loss (L) probabilities when Team A has declared in the third innings with a specified target and a specified number of overs remaining. The SETAL probabilities are given in parentheses.

estimation of parameters. We consider the 49 test matches between ICC teams that were played between April 1, 2012 and April 30, 2013. In these matches, we examine the third innings scorecard, and upon the completion of each over, we ask whether our declaration rule ought to have been invoked. There were 13 matches where actual declarations took place in the third innings. Of the remaining 36 matches where there were no actual third innings declarations, our decision rules also did not invoke a declaration. This suggests that teams are making good decisions with respect to declaration although they may not be doing so at optimal times. From the 13 matches where declarations occurred in the third innings, two of these matches were rained out before the fourth innings began. This reduces our study to 11 matches of interest. In the 11 matches of interest, we simulated match results according to our declaration rule. If our declaration preceded the actual declaration, we simulated fourth innings batting using Table 6 from our point of declaration. If our declaration did not precede the actual declaration, we simulated third innings batting using Table 2 from the actual point of declaration until our rule prescribed a declaration (if at all), and then from that juncture, simulated fourth innings data. In summary, we compare how the matches actually turned out with the way they would have

turned out (via simulation) had our decision rules been followed.

$\hat{p}_{-1}$	$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$	$\hat{p}_5$	$\hat{p}_6$
0.016	0.743	0.128	0.035	0.010	0.065	0.000	0.003

Table 6: Sample proportions corresponding to the parameters $p_{-1}, \ldots, p_6$ for fourth innings batting using the original 134 matches.

In Table 7 and Table 8, we provide a summary of the 11 match results based on 1000 simulations per match. The simulation in Table 7 used the standard criterion (1) where wins and draws are both valued. The simulation in Table 8 used criterion (2) where Team A is desperate and has a win at all costs perspective. In both tables, we provide simulation results where Team B is considered to be a relatively average team in comparison to Team A. We also consider the case where Team B is considered to be either relatively weak or relatively strong in comparison to Team A. The determination of weak or strong is obtained from the ICC Test rankings www.cricketworld4u.com/team-ranking.php.

Actual Results						Simulated Results					
Date	TA	TB	Target	OR	Result	TB	Target	OR	P(W)	P(D)	P(L)
Apr 15	Aus	WI	214	59	D	weak	233	55	0.10	0.90	0.00
						average	268	49	0.02	0.98	0.00
Jun 22	SL	Pak	509	195	W	average	-	-	-	-	-
Jun 30	Pak	SL	260	37	D	weak	218	43	0.02	0.98	0.00
						average	239	40	0.01	0.99	0.00
Jul 08	Pak	SL	269	71	D	weak	261	74	0.36	0.64	0.00
						average	290	65	0.11	0.89	0.00
Aug 02	SA	Eng	252	39	D	average	241	40	0.01	0.99	0.00
Nov 25	NZ	SL	363	106	W	average	358	107	0.27	0.39	0.34
Feb 01	SA	Pak	480	255	W	weak	-	-	-	-	-
Mar 08	SL	Ban	268	22	D	weak	-	-	-	-	-
Mar 22	NZ	Eng	481	143	D	average	472	145	0.44	0.51	0.05
Apr 17	Zim	Ban	483	236	W	average	-	-	-	-	-
Apr 25	Ban	Zim	401	134	W	average	398	135	0.32	0.08	0.60

Table 7: Comparison of actual results versus simulated results using our declaration rules and according to the standard criterion (1). Abbreviated column headings are TA (Team A), TB (Team B) and OR (Overs Remaining). Team B is characterized as relatively average, weak or strong in comparison to Team A. The matches from June 22/12, Feb 1/13, Mar 8/13 and Apr 17/13 were not simulated because the optimal number of overs exceeded 180.

From Table 7, we first observe that in 6 of the 7 matches, our declaration took place slightly earlier than the actual declaration. In the April 15 match where Australia actually declared

	Actual Results						Simulated Results				
Date	TA	TB	Target	OR	Result	TB	Target	OR	P(W)	P(D)	P(L)
Apr 15	Aus	WI	214	59	D	weak	173	68	0.23	0.51	0.26
						average	194	64	0.08	0.46	0.46
Jun 22	SL	Pak	509	195	W	average	-	-	-	-	-
Jun 30	Pak	SL	260	37	D	weak	160	55	0.10	0.85	0.05
						average	165	53	0.03	0.59	0.38
Jul 08	Pak	SL	269	71	D	weak	212	80	0.41	0.47	0.12
						average	243	77	0.19	0.51	0.30
Aug 02	SA	Eng	252	39	D	average	165	58	0.03	0.29	0.68
Nov 25	NZ	SL	363	106	W	average	339	114	0.31	0.28	0.41
Feb 01	SA	Pak	480	255	W	weak	-	-	-	-	-
Mar 08	SL	Ban	268	22	D	weak	-	-	-	-	-
Mar 22	NZ	Eng	481	143	D	average	459	148	0.48	0.40	0.12
Apr 17	Zim	Ban	483	236	W	average	-	-	-	-	-
Apr 25	Ban	Zim	401	134	W	average	379	140	0.36	0.03	0.61

Table 8: Comparison of actual results versus simulated results using our declaration rules and according to the "must win" criterion (2). Abbreviated column headings are TA (Team A), TB (Team B) and OR (Overs Remaining). Team B is characterized as relatively average, weak or strong in comparison to Team A. The matches from June 22/12, Feb 1/13, Mar 8/13 and Apr 17/13 were not simulated because the optimal number of overs exceeded 180.

earlier than what our decision rules stipulate, we note that Australia's declaration is premature compared to the historical declaration decisions in Figure 1. As expected, we also observe that when Team B is a weaker opponent, then the declaration occurs earlier than if Team B is an average opponent. When looking at the simulation probabilities for wins, draws and losses, it appears that the outcomes using our declaration rules are not in conflict with the actual outcomes.

From Table 8, we observe that Team A declares earlier than in Table 7. This is anticipated since in Table 8, Team A is desperate to win. Consequently, the win and loss probabilities in Table 8 are larger than those in Table 7.

7   DISCUSSION

The determination of when to declare in test cricket is an important problem which affects the outcomes of matches. In this paper, we provide specific guidelines (Table 4) whether teams should declare at various stages of matches.

Relating our optimality results to current practice, we have the following summary guidelines.

Teams that declare with less than 90 overs remaining (which is an entire day of batting), are behaving cautiously and should actually declare earlier. In particular, teams wait far too long when it is essential that they win the match. Although waiting longer always reduces the chance of losing, it may also decrease the chance of winning. SETAL also remark that teams are generally cautious when declaring. However, for optimal decision making, teams that declare earlier (say, with more than 135 overs or 1.5 days remaining), should actually wait a little longer to declare.

We add a caveat with respect to our results. The run scoring characteristics of Team B in the fourth innings are based on standard batting behaviour. It is possible that teams become cautious in situations where they believe that winning is impossible, and they instead play for a draw. However, our Table 4 provides declaration guidelines at the tipping point where it turns advantageous for Team A to declare. These are not extreme situations, and we suggest that Team B is likely to employ standard batting behaviour in these circumstances.

In sport, teams that adopt improved strategies are able to gain a competitive edge (see Lewis, 2003). It is hoped that papers such as this will help promote the adoption of analytics in cricket. At the present time, the use of cricket analytics appears to trail many of the popular professional sports.

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