Math 398 - Homework 1

1. Let $\mathbf{c} : (-1, \infty) \to \mathbb{R}^2$ be given by

$$\mathbf{c}(t) = \left(\frac{3at}{1+t^3}, \frac{3at^2}{1+t^3} \right).$$

Show that

(a) For $t = 0$, $\mathbf{c}$ is tangent to the x axis.

(b) As $t \to \infty$, $\mathbf{c}(t) \to (0, 0)$ and $\mathbf{c}'(t) \to (0, 0)$.

(c) As $t \to -1$ (that is, take the curve with the opposite orientation), the curve and its tangent approach the line $x + y + a = 0$.

(d) Draw the curve in the xy plane. The curve is part of what is called the *folium of Decartes*.

2. Consider a planar curve $\alpha : I \to \mathbb{R}^2$ parametrized by arc length (the curve has unit-speed). Assume that $k(s) \neq 0$, for all $s \in I$. In this situation, the curve

$$\beta(s) = \alpha(s) + \frac{1}{k(s)}\mathbf{n}(s), \qquad s \in I,$$

is called the *evolute* of α at s.

(a) Show that the tangent at s of the evolute of α is normal to α at s.

(b) Draw a figure that shows a curve and its evolute.