Math 398 - Homework 2

1. Let $\mathbf{c} : I \to \mathbb{R}^3$ be a regular parametrized curve with nonzero curvature everywhere and arc length as parameter. Let

$$\mathbf{r}(s,v) = \mathbf{c}(s) + r(\mathbf{n}(s)\cos v + \mathbf{b}(s)\sin v), \qquad r = \text{const.} \neq 0, s \in I,$$

be a parametrized surface (the *tube* of radius r around $\mathbf{c}$), where $\mathbf{n}$ is the principal normal vector and $\mathbf{b}$ is the binormal vector of $\mathbf{c}$. Show that, when $\mathbf{r}$ is regular, its unit normal vector is

$$\mathbf{N}(s,v) = -(\mathbf{n}(s)\cos v + \mathbf{b}(s)\sin v).$$

2. Given the parametrized surface

$$\mathbf{f}(u,v) = (u\cos v, u\sin v, \log(\cos v) + u), \qquad -\frac{\pi}{2} < v < \frac{\pi}{2},$$

show that the two curves $\mathbf{f}(u_1, v)$, $\mathbf{f}(u_2, v)$, where u_1 and u_2 are two fixed values, determine segments of equal lengths on all curves $\mathbf{f}(u, \text{const.})$.