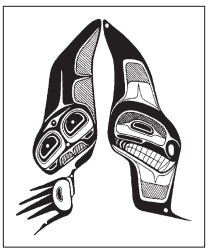

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Calculus 12: The Ultimate Pre-calculus Course

Abstract

For many students, the first calculus course is the biggest challenge at the beginning of their post-secondary education. The paper discusses questions related to the preparation of secondary school students for this challenge. In particular, the authors are interested in the role of Calculus 12 in the light of the recent changes in the curriculum of Principles of Math 12. Analyses conducted in recent years at both Simon Fraser University and the University of British Columbia indicate that Calculus 12 is the best preparation for university level calculus courses. The paper looks into why this is so, and how awareness of this influences teaching calculus both at both the university and secondary school levels.

1. Introduction

The main question that we are interested in is the following. What is the role of Calculus 12 as a link between secondary school mathematics and the first calculus course at the post-secondary level? In an attempt to answer this question, we give the definition of Calculus 12 as the Ministry of Education gives it, together with various practices in teaching Calculus 12 and different ways that post-secondary institutions in British Columbia handle freshmen who come with a secondary school calculus course. Our objective is to give one possible view of the current situation and to offer potential answers rather than to look for definite solutions of the problem of harmonizing teaching of secondary and post-secondary mathematics.

The first year calculus course is the biggest academic challenge for many students at the beginning of their post-secondary education. The course is usually topic-packed and fast-paced, and taught in large lecture halls or in a multiple section setting. The typical course covers limits, continuity, differentiation and its application. The type of applications and the level of complexity depend on the stream in which the course is. For example, a calculus course for engineering and science students might include an epsilon–delta definition of continuity, while a course for business students might not.

Common to all calculus courses at this level is the expectation that the incoming students have a clear idea what a function is and a fair knowledge about several types of functions, their basic properties, and their graphs. This includes linear and quadratic functions, polynomials in general, rational, exponential, logarithmic, and trigonometric functions. Students' confidence in manipulating algebraic expressions and familiarity with elementary geometry is also expected. In BC, these topics are covered in Principles of Mathematics 11 and Principles of Mathematics 12. It should be clear that the expectations are quite high and to assure that those expectations are met, post-secondary institutions require a relatively high mark in Principles of Mathematics 12. For example, Simon Fraser University requires at least a B grade in Principles of Mathematics 12 for all first year calculus courses. Many institutions offer pre-calculus courses. These courses give an overview about functions and their properties, with attention to the families of functions that are listed earlier in this paragraph.

Therefore, for success in a first calculus course, an incoming student does not need any previous experience with calculus. In other words, a secondary school calculus course is not a necessary condition for understanding all the material taught in the university course. This said, a couple of questions arise. The first question is, "Why do secondary school students take Calculus 12 if they neither need this course for a prerequisite nor get a university credit for it?" The other question goes in the opposite direction. Faced with the reality that some incoming students have previous calculus experience, how can universities and colleges best organize the teaching of calculus courses? In this article, we offer possible answers to both questions.

The paper is organized in the following way.

In Section 2, we give the description of Calculus 12 as the Ministry of Education sets it. Also, in this section we compare the list of topics in a standard pre-calculus course with the curricula of Principles of Mathematics 11 and 12. We use segments from two recent surveys to illustrate the role of the Principles of Mathematics pathway in preparing secondary students for calculus.

In Section 3, we describe how Calculus 12 has been advertised in BC secondary schools and we list different practices in teaching Calculus 12. In this section, we also describe the Calculus Challenge Exam.

In Section 4, we give data obtained by the University of British Columbia and Simon Fraser University about students' performance in first year calculus courses. We list current offerings of calculus courses, stressing the differences between courses for students with previous calculus experience and courses for those without it. We also include information about practices at a few BC colleges.

2. Calculus 12 – A Member of the Pack

The structure of secondary mathematics courses in British Columbia is given by Figure 1. In the BC curriculum model, math topics are re-visited in successive grades as students learn the material in greater depth and to an enhanced extent.

In the present form, Calculus 12 has been offered in BC secondary schools since the year 2000. As it is shown in figure 1, Calculus 12 is an addition to the Principles of Mathematics pathway. It is intended for "students who have completed (or are concurrently taking) Principles of Mathematics 12 or who have completed an equivalent college preparatory course that includes algebra, geometry, and trigonometry." (Source: http://www.bced.gov.bc.ca/irp/math1012/mapath.htm)

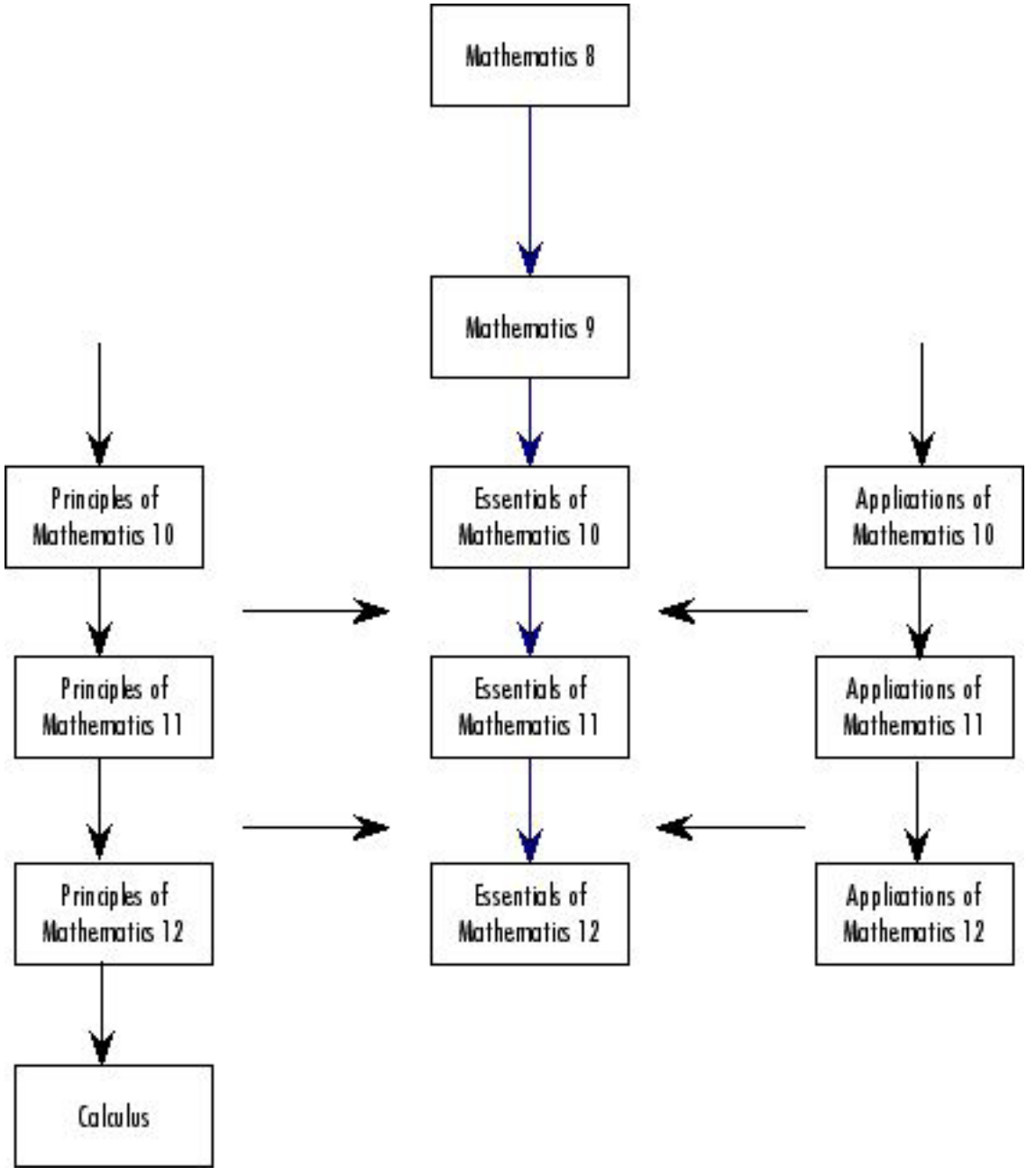

Figure 1

Source: http://www.bced.gov.bc.ca/irp/math1012/marat.htm

The course has been developed under the assumption that teachers have 100 instructional hours available to them. Table 1 shows the estimated instructional time in hours by curriculum topics as suggested by the BC Ministry of Education. As a comparison, in the most right column of Table 1 we give estimated instructional times for the same or similar topics in SFU Math 151 – Differential Calculus and Math 152 – Integral Calculus (Summer Semester 2004). We note that Math 151 and Math 152 each run for thirteen weeks, with three hours of lectures per week. In the 2004 summer semester, about 80% of the instructional time in Math 151 was spent on the topics that were listed by the Ministry of Education as topics in Calculus 12. It is important to observe that the Calculus 12 "teachers may freely adjust the instructional time to meet their students' diverse needs." (Source: http:// www.bced.gov.bc.ca/rp/math1012/calc12est.html .)

Topic	Calculus 12	SFU Math 151/152
Functions, Graphs, and Limits	10 – 15 hours	4
The Derivative (Concept and Interpretations)	10 - 15	2
The Derivative (Computing Derivatives)	15 - 20	6
Applications of Derivatives (Derivatives and the Graph of the Function)	15 - 20	8
Applications of Derivatives (Applied Problems)	20 - 25	7
Antidifferentiation (Recovering Functions from their Derivatives)	5 - 10	10
Antidifferentiation (Applications of Antidifferentiation)	10 - 15	12

Table 1

Source: http://www.bced.gov.bc.ca/irp/math1012/calc12est.htm

A comparison of the instructional time for the topics in the first 5 rows in Table 1, which are common for Calculus 12 and Math 151, suggests that students in Calculus 12 have been given more than double the instructional time than students in Math 151 to learn the same material. So, why does Calculus 12 not transfer at least as the first (differential) university calculus course?

It is important to note that the curriculum of the first university calculus course does not require any previous experience with calculus and that a fair knowledge of topics from Principles of Mathematics 11 and 12 should be enough to assure students' success in the calculus course. We illustrate this fact by Table 2 where we compare topics from Math 100, a standard pre-calculus course offered by Simon Fraser University, and topics from Principles of Mathematics 11 and 12.

Topic	Math 100	Math 12	Math 11
Graphs, Functions and Models	+	+	+
Polynomial and Rational Functions	+		+
Exponential and Logarithmic Functions	+	+	
Trigonometric Functions	+	+	
Analytic Geometry	+	+	+

Table 2

(The source for topics in SFU Math 100 was the course outline posted on http://www.math.sfu.ca/courses/next/math100.shtml . The source for topics in Principles of Mathematics 11 and 12 was http://www.bced.gov.bc.ca/irp/math1012/mathtoc.htm).

Ms. Rose Albiston, a math teacher from Terry Fox Secondary, Port Coquitlam, BC, compares the current secondary school math curriculum with the curriculum before 2000 in the following way.

> *… the high school program … is not as much of a 'spiral' in presentation as it used to be before the current program. In the old program geometry was introduced in Math 8 and revisited subsequently in 9, 10, 11 and 12. In the current program the students 'officially' get it (possibly) in Grade 8 (middle school in Coquitlam) and again in Grade 11 only. Similarly in the old program trigonometry was introduced in Grade 9 and developed through Math 10, 11 and 12 as opposed to the students getting trigonometry only in Math 10 and 12 in the current. In addition 'officially' topics previously not even part of the Grade 12 course have been relegated to Math 11 where the students are anything but ready to do justice to the topics of rational, radical and absolute value inequalities and analytic geometry (with virtually no geometry background). In addition advanced algebra, which was part of Math 12, was moved down to Math 11 to make room for the finite math topics at the Grade 12 level. In other words Math 11 and 12 combined do not adequately develop significant portions of the algebra and geometry concepts that would be very important for a student to be comfortable in a post secondary calculus course. (Not to mention that some topics have simply been left out...factoring of the sum and difference of cubes, inverse variation...)"*
>
> *(Rose Albistone, Personal communication)*

In "Mathematics Proficiency for Post-Secondary Mathematics/Statistics Courses,

Project Report" by L. G. Neufeld (1999), the author gives the ranking of the order of importance of the proficiency categories for calculus. In Table 3 we give some of the results obtained by Neufeld.

Proficiency Category Description	Overall Importance Rating (Out of 4)
Understand and use the Function Concept	4.00
Understand and use Polynomial Expressions	3.96
Understand and use Exponential Expressions	3.96
Understand and use Straight Line and Linear Functions	3.96
Solve Equations and Inequalities	3.92
Understand and use Circular Trigonometric Functions	3.92
Understand and use Rational Expressions	3.83
Understand and use Triangle Trigonometry	3.83
Understand and use the Quadratic Function	3.83
Understand and use the Logarithmic Function	3.79
Understand and use Radical Expressions	3.75
Understand and use the Geometry of Lines and Points	3.50
Understand and use Polynomial Functions	3.42
Understand and use Quadratic Relations	3.29
Understand and use Sequences and Series	2.75
Understand and use the Geometry of Circles	2.65
Understand and use some Concepts of the Calculus	1.06

Table 3

Source:
http://members.shaw.ca/bccupms/document/mathematics%20Proficiencies%20Project.pdf (p. 17)

We note that Neufeld's 1999 Report was one the documents that determined the curriculum changes of the secondary mathematical courses that were made in 2000. Consequently, since "one of the primary purposes of Principles of Mathematics will be to develop the formalism students will need to continue on with the study of calculus" the current lists of topics in Principles of Mathematics 11 and 12 mirror in a great extent Neufeld's findings. (http://ww.bced.gov.bc.ca/irp/math1012/mapath.htm)

As a curiosity we note that "Understand and use some Concepts of the Calculus" has a rating 1.06 (out of 4) which brings us to the following question. If Calculus 12 does not transfer to the first year calculus course and it is not necessary for success in the university course, what motivates students to take it and, as we will see later, universities to distinguish freshmen with Calculus 12 from those without it?

A possible answer to this question is offered by the "Western and Northern Canadian Protocol, Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Preliminary Report and Findings" prepared by System Improvement Group Alberta Education and distributed on April 30, 2005.

> *Pathway 1 has too many outcomes to be taught in one 5-credit course per year, within the number of hours allocated to 5-credit courses. It has 153 outcomes, as compared to the 115 outcomes currently included in Principles of Mathematics pathway, which itself is considered too packed. Other options will need to be explored for delivering these outcomes. One possibility is to have students aiming for the Science take two mathematics courses in one or more of the high school grades, and the Science major areas would ask for completion of both grade 12 courses for admission. Another possibility is o divide the outcomes into those that will be taught in high school and those that will be taught in the first year post-secondary year. (p. 48)*

> *Pathway 1 – If an outcome was identified as "master" by at least 50% of "Science" (calculus-based) respondents and/or as "master" or "expose" by at least 70% of such respondents, it was included herein. (p.4)*

In our opinion, Calculus 12 follows the spirit of the above quote. This course is especially useful for secondary students because it gives them a breathing space, an opportunity to review, or to learn, the pre-calculus material and, at the same time, links them directly with the material that they will learn in the post-secondary institution of their choice.

3. Calculus 12 – In Practice

In this section we describe how Calculus 12 has been advertised in the British Columbia secondary schools. We list different practices in teaching Calculus 12 and we offer a few reasons why students take Calculus 12. Finally, we describe the Calculus Challenge Exam.

The Ministry of Education proposes the outcome for students completing Calculus 12 in the following way.

> *Students taking Calculus 12 should be prepared to write the UBC - SFU - UVic - UNBC Challenge Examination if they choose to do so. (…) Some schools may choose to develop articulation agreements with their local colleges. Students under these agreements may receive credit for first-term calculus (depending upon the particular agreement).*
>
> *(Source: http://www.bced.gov.bc.ca/irp/math1012/mapath.htm)*

Thus the intention of the creators of the current concept of Calculus 12 in British Columbia was to allow secondary school students to obtain a university credit for first-term calculus through the Calculus Challenge Exam or through articulation agreements.

Our research of the yearly calendars and course outlines shows that, in the BC secondary school system, Calculus 12 is advertised mostly as a transition course, a course that prepares students for the first-term post-secondary calculus course. In other words, Calculus 12 *is* advertised as a "pre-calculus" course. We support this claim with the following two quotes.

Calculus [12] will introduce the student to the fundamentals of differentiation and integration along with applications. Topics include graphing, maxima and minima, related rates, area, volumes, and exponential functions. This course is a good introduction to university level calculus.

(Source: http://cariboo.sd41.bc.ca/departments/math/index.html)

This course [Calculus 12] is intended as an introduction to calculus for students who intend to take calculus at university. It is not an "advanced placement" course designed to replace first year calculus. It is rather intended to show you what calculus is, in order to ease your transition into first year calculus."

(Source: http://magee.vsb.bc.ca/dsheldan/calculus12/pdf/calculus12outline.pdf)

In our opinion there are at least two reasons for this discrepancy between the outcome suggested by the Ministry ("… should be prepared … may receive credit …") and the practice ("… a good introduction … to easy your transition…") One is that some secondary schools offer so-called AP (Advanced Placement) Calculus, a course that under certain grade conditions all BC universities transfer as a first year calculus.

(Note: For more about the Advanced Placement program see http://en.wikipedia.org/wiki/Advanced_Placement_Program)

Another and, for the authors of this note, more important reason is that schools recognize that an additional math course helps students to achieve the level of mathematical knowledge and maturity necessary for success in post-secondary math courses.

Currently Calculus 12 is taught as a mixture of a remedial course and a preview course for first year Calculus at University/ College. The degree of mixing these two extremes depends on the school, the instructor, and the particular group of students.

We categorize teachers of Calculus 12 into four groups when considering the content of their Calculus 12 courses. (This categorization is based on the experience of the second author as a Calculus 12 teacher and his communication with other Calculus 12 instructors.)

- Teachers who generally follow Ministry of Education guidelines
  At the same time they do much more filling in of gaps from the secondary school program.
- Teachers who spend little time on any calculus topics and most of the time on the pre-calculus topics such as lines, functions and graphs.
  Since the pre-calculus skills learned here are asked upon throughout first year college calculus, their mastery is expected.
- Teachers who focus on all the fundamentals of calculus (differentiation and integration) and remove the aspects of algebra, such as simplifying, that increase the difficulty of the questions without adding to the understanding of calculus.
  The algebra that is taught/reviewed includes the common techniques embedded in the solving of calculus, such as use of the conjugates. The algebra in simplification is not stressed as the students often do not see and understand the difference in their answers.
- Teachers who teach Calculus 12 as if it were a first year post-secondary calculus course

One of the implicit factors facing all Calculus 12 teachers is that there is more to teach than there is time allowed. A slower pace brings most of the class to a better understanding of the topics covered, but not as many topics can be covered. The alternative has most, or all of the topics covered, but numerous students not completely understanding. It should be noted that students taking Calculus 12

are good but usually not the elite math students as the latter are most often taking AP Calculus. Most teachers aim to prepare their students for post-secondary Calculus. What they teach, or do not teach can be justified, as there is little time and so much more that could be taught.

Calculus 12 is a unique secondary school course: it is rarely taken by students not going on to university/college, yet it gives no assistance in gaining entry into post secondary institutions in British Columbia. (We note that students from British Columbia applying at Canadian universities outside British Columbia are often asked to have Calculus 12 as one of admission requirements.)

Students who take calculus at the secondary school level are preparing themselves for first year calculus at post-secondary schools. These students are all typically good at mathematics, having obtained A's and B's in Principle of Mathematics 11 and 12. Some of them have already completed Principle of Mathematics 12, while others are taking Principle of Mathematics 12 concurrently with Calculus 12. It is our experience that all of the students are familiar with the fact that BC universities will not look at the course as an entrance requirement. When one considers the pressure of getting high marks on numerous Grade 12 provincial exam courses and the difficulty of Calculus 12, a student should have good reasons for taking Calculus 12. What are these reasons? We suggest a few of them.

Secondary school students have heard that the first year university calculus course is extremely difficult and unlike anything they have yet learned. Topics from Mathematics 8 to Principles of Mathematics 12 have little connection to the fundamentals of calculus. Students who go through one of the mathematics streams in secondary school have had the same topics slowly taught to them over 5 years. Mathematics 8 teaches the basic skills that will be built upon in Mathematics 9 to 12. For example, the trigonometry covered in Principle of Mathematics 12 is needed in calculus, but the basics of trigonometry are first taught in Principles of Mathematics 10. Manipulation of exponents and powers is taught in Grade 8 and expanded upon in later years to include radicals and other manipulation methods such as conjugates. Each of the mathematical skills a student gains from Mathematics 8 to 12 is taught on an overlapping gradual agenda. Calculus on the other hand is a completely new mathematical concept and students use Calculus 12 to discover some of the mystery that surrounds it.

First year university calculus students often regret not taking Calculus 12 in secondary school. (This claim is based on the numerous conversations that both authors have had with their current and former students.) Those students felt they were at a disadvantage learning calculus for the first time. Having heard these regrets, secondary school students are motivated to take Calculus 12.

Some of the best secondary students want to get as high a mark as possible in the first year of university. By taking and working hard in Calculus 12, students increase their chance of maintaining a high GPA during the first year, which enables them to receive financial rewards and bursaries.

We complete this section with some facts regarding the Calculus Challenge Examination.

(Source: http://www.math.sfu.ca/outreach/schools/challenge)

The exam is organized jointly by Simon Fraser University, the University of British Columbia, the University of Northern British Columbia, and the University of Victoria. It is based on the Calculus 12 curriculum and it could be taken by students who have studied calculus in school and have not yet started college or university. Only one attempt is permitted. Writers scoring over 50% are entitled to request credit for the first term of calculus at any of the four universities listed above. The exam score will be shown as a course grade from the university they attend. We note that the choice of claiming the credit and grade is up to the student. Students who pass have the option of ignoring their exam score and taking first-year calculus for credit instead.

The cost of the exam in 2006 is $ 88. About 200 students took the exam in 2004.

4. Calculus 12 – The Day After

In this section, we give data obtained by the University of British Columbia and Simon Fraser University about students' performance in first year calculus courses. We list current offerings of calculus courses, stressing the differences between courses for students with previous calculus experience and those without it. We include information about practices at a few BC colleges.

In the 1993 BCAMT Fall Conference, Dr. George Bluman presented the following table. (Source: G. Phyllips and C. Koe, *Making Time for Mathematics, Vector* 35:2, Spring 1994, pp. 18-23.)

Year	Number enrolled in first-year calculus at UBC	Failure rate
1976	1713	20%
1977	1599	29%
1991	1701	15%
1992	1622	10%

Table 4

Bluman explained the dramatic drop in the failure rate from the mid seventies to the early nineties. "In the '70s, less than 10% of the (secondary) students were taking one semester of calculus, and less than 10% were taking a year-long calculus course. In the '90s, over 65% have at least one semester of calculus in addition to Mathematics 12."

In September 2004 at Simon Fraser University, 946 BC secondary students took one of the first semester calculus courses. Among those students, 191 came with Calculus 12, which made up about 20% of this segment of the freshmen population. Table 5 gives the percentage breakdown by class and grade of B.C. high schools graduates that were enrolled in one of the first semester calculus courses at SFU in fall 2004. We note that Table 5 supports the claim that the majority of B.C. secondary school students who took Calculus 12 obtained an A in Math 12. (The data in Table 5 and Table 6 is provided to the authors by Dr. Malgorzata Dubiel, Simon Fraser University.)

Math 12	Calculus 12	%
A	Yes	16.2
A	No	44.7
B	Yes	3.6
B	No	32.6

Table 5

The performance of the SFU Math 151 class is summarized in the following table. (SFU Math 151 – Calculus I is a class for students with intended major in mathematics, computing science or engineering.)

Math 12	Calculus 12	Math 11	
		A	B
A	Yes	3.23	2.67
A	No	2.63	1.92
B	Yes	2.22	2
B	No	1.98	1.43

Table 6

For example, under column A, the grade point average in Math 151 for students who obtained an A grade in both Principles of Mathematics 12 and Principles of Mathematics 11and who took Calculus 12 was 3.23 (out of 4). We note that, for this particular group of students, there was a significant difference in performance in Math 151 depending on two parameters.

First, we observe that students with no Calculus 12 underperformed in Math 151 comparing with students with the same grades in Math 12 and Math 11 but with Calculus 12. In Table 7 we show the relative changes in the average marks in Math 151 for students with the same grades in Math 12 and Math 11 but with and without Calculus 12.

Math 12	Calculus 12	Math 11	
		A	B
A	Yes	1	1
A	No	0.81	0.72
B	Yes	1	1
B	No	0.89	0.72

Table 7

Secondly, Table 6 suggests the importance of Math 11 in preparation for a university calculus course. In Table 8 we show the relative changes in the average marks in Math 151 for students with the same grades in Math 12 and the same Yes/No status in Calculus 12 but with different marks in Math 11.

Math 12	Calculus 12	Math 11	
		A	B
A	Yes	1	0.83
A	No	1	0.73
B	Yes	1	0.90
B	No	1	0.72

Table 8

Faced with the fact that a growing number of freshmen in calculus classes have previous experience with calculus, the University of British Columbia and Simon Fraser University introduced separate calculus classes for the two groups.

In fall 2001, UBC introduced new calculus courses for students without a previous calculus course. The main change was that the newly created courses had an additional credit/weekly lecture hour. The following example, a Calendar description for UBC Math 100 and Math 180, illustrates the change.

MATH 100 (3) **Differential Calculus with Applications to Physical Sciences and Engineering**: Derivatives of elementary functions. Applications and modeling: graphing, optimization. [3-0-0]
Prerequisite: A score of 64% or higher in Principles of Mathematics 12 and high-school calculus.

MATH 180 (4) **Differential Calculus with Physical Applications**:
Topics as for Math 100; intended for students with no previous knowledge of Calculus. Not for credit for students with High School Calculus, AP Calculus AB, AP Calculus BC, or a passing score on the UBC-SFU-UVIC-UNBC Calculus Challenge Examination. [4-0-0]
Prerequisite: A score of 64% or higher in one of Math 099, Principles of Mathematics 12.

(Source: http://students.ubc.ca/calendar/courses.cfm?code=MATH)

Both classes write the same final exam, and the rest of the courses in the calculus sequence are same for the both groups.

The following table compares results of the two classes in December 2004 for students from the graduating classes of BC secondary schools. Results for 2003 are given in brackets.

	Math 100	Math 180
# of students	839 (972)	460 (423)
% with A standings	37 (27)	18 (9)
% passing	93 (87)	76 (74)
Average school mark	91 (91)	87 (86)
	↓	↓
Average UBC mark	73 (68)	62 (57)

Table 9

(Source: http://www.math.ubc.ca/Schools/FirstYearcalculus/index.shtml)

We note that the results for previous years (2001 – 2003) are similar to those given in Table 9 and that, together, they confirm that generally speaking, students with a secondary school calculus course perform better in the first university course than students without a secondary school calculus course, even if the latter group gets an additional weekly hour of lectures.

In 2005, Simon Fraser University decided to go with a similar concept and to offer Math150, a 4-credit calculus course for freshmen with no previous calculus experience. In the rational for introduction of Math 150 it was said that the proposed course would be an "alternate choice for Math 151, intended for students with a somewhat weaker high school background".

It is important to repeat that the curriculum of the first university calculus course does not require any previous experience with calculus and that a fair knowledge of topics from Math 12, and as we have seen in Table 2, from Math 11 should be enough to assure students' success in the calculus course. In our opinion, the fact that two major BC universities opted to offer an additional weekly hour for students with Math 11 and Math 12, but with no Calculus 12, shows that there is a gap between the projected and actual outcomes in math knowledge for graduates from B.C. secondary schools.

Community and university colleges play a significant role in the post-secondary education system in British Columbia. Traditionally, the colleges offer courses that are equivalent to BC Math 11, BC Math 12, pre-calculus courses, and a full array of university calculus courses. Typically, all of those courses are taught with 4 hours of lectures per week in classes with less than 40 students.

Mr. Wesley Snider, a math instructor at Douglas College, New Westminster, B.C., describes the current state of teaching calculus classes at Douglas.

> *It is not feasible for us to have a different stream for students with Calc 12. I presume SFU is thinking of adding an additional hour for students without Calc 12. All of our Calculus I (for science) students have 4 hours of lecture plus another 2 hours of tutorial each week. Even at that the success rates are not very good. Therefore we cannot increase the time any longer for the weaker students nor decrease the time for the stronger students (given the success rates). The only discrimination I can see at the moment may be in terms of prerequisites for Calc I. We currently take students with a Math 12 A or B grade. It could be that in future we take a Math 12 A, or a Math 12 B plus Calc 12 B (or C). I think these discussions will be coming up shortly here at Douglas.*
>
> *(Wesley Snider, personal communication)*

On the other hand, Vancouver Community College (VVC) in the sequence "Upgrading Courses," offers a couple of courses that together match Calculus 12. The objectives of these two courses are given in the following way.

> *Math 096 and Math 097 are introductory calculus courses designed to ease the transition from Math 12 to 1st year calculus at college/university. Students completing both Math 096 and 097 are eligible to write BC University calculus Challenge Examination. Students who pass this exam and go on to a BC University may claim credit and exemption from the first semester of university calculus.*
>
> *(The course content of VVC Math 096)… covers a solid review of Math 12 topics required to succeed in 1st semester Calculus as well as the fundamentals of differential calculus: the limit concept, the concept of continuity, the derivative and rate of change, basic differentiation rules, derivatives of algebraic functions, maxima and minima, applied optimization problems and curve sketching.*
>
> Source: http://upgrading.vcc.ca/math/coursedes097.cfm

We note that VVC offers a standard pre-calculus course Math 1020. We quote the course description for Math1020 as a comparison with the course content for VCC Math 096.

> *Pre-calculus is intended for students planning to take calculus for science, business, commerce and social science programs. Emphasis is placed on the extensive study of polynomial, rational, logarithmic, exponential, trigonometric functions, their inverse and applications. The objective of the course is to provide a solid foundation for the development of calculus.*
>
> *(Source: http://www.vcc.ca/programs/detail-course.cfm?WPGM_PROGRAM_ID-155&WC2P_ COURSE_ID =2682&DIVISIONID=16)*
>
> *As a curiosity we mention that there is a college in British Columbia that lists Calculus 12 as a recommended course for a one hundred level physics course.*
>
> *(Source: http://web.mala.bc.ca/hearnd/Courses/Phys121/Course%20Outline/default.htm)*

We conclude this section with a quote from Dr. Lin Hammill, a math instructor from Kwantlen University College, Richmond, BC. Dr. Hammill describes her experience with students coming to Kwantlen with Calculus 12.

> *Now, what do I think of the high school calculus? My experience has not been very positive. I find that the quality of learning varies greatly depending, it seems, on the individual teacher. Overall, I find that the students have been taught differentiation formulae and techniques and are able to find critical points. However, they really do not understand what a derivative is, what critical points really are and how to use the information supplied to solve problems (as opposed to completing exercises). They come in over-confident and so many of them do not settle down to work right away and this can be very damaging to them. They sometimes find it difficult to catch up once they realize that they actually do not know everything already. I would prefer they have no calculus at all, but have a much better grounding in the basics: functions, graphs, solving equations (without the use of a graphing calculator) and doing multi-step problem solving. This will serve them far better than a catalogue of differentiation formulae. On the other hand, since only the more able students are likely to opt for calculus in high school, perhaps this is a way to screen for those who are more likely to do well in calculus.*
>
> *(Lin Hammill, personal communication)*

We note that Dr. Hammill's comment is in the line with Neufeld's findings and recommendations given by "Western and Northern Canadian Protocol, Consultation with Pos-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Preliminary Report and Findings". At the same time, the experience from the University of British Columbia and Simon Fraser University shows that the majority of students with Calculus 12 perform better in a first post-secondary than their peers without Calculus 12.

5. Conclusion

In British Columbia secondary schools, Calculus 12 is advertised and taught as a course that will ease students' transition into first year university calculus. Calculus 12 is taken mostly by students planning to continue their schooling at post-secondary institutions. The University of British Columbia and Simon Fraser University have introduced separate courses for students with and without Calculus 12.

The main benefit for students taking Calculus 12 is that an additional math course in Grade 12 helps them to reach the mathematical maturity level needed for success in a post-secondary calculus course. This is obtained by reviewing the material learned in other secondary school math classes, filling in of gaps from the secondary program, and learning the basics of calculus.

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