1 Sample Exam

1.1 BCIT Challange Exam for OPMT 5701

1. Find $AB = C$ If

$$A = \begin{bmatrix} 5 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 4 & 3 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}$$

2. Find the determinant by LAPLACE Expansion of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

3. Find the inverse of A using the formula $A^{-1} = \frac{1}{|A|} Adj A$ if

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$$

4. Use Cramer's Rule to solve the following system for x and y

$$4x + 3y = 28$$
$$2x + 5y = 42$$

5. Use Cramer's Rule to find x and y for

$$4x + 8y = 96$$
$$6x - 3y = 12$$

6. Find dy/dx if

$$y = 3x^{1/4} - 12^2 - 8x^{-3/4}$$

7. Find dy/dx if

$$y = (x+1)(x-1)$$

8. Find dy/dx if

$$y = (x^2 - 4)(5x^3 + x)$$

9. Find dy/dx if

$$y = \frac{\sqrt{x}\left(1 - x^2\right)}{x}$$

10. Find dy/dx if

$$y = (x^2 + 9)\sqrt{x^2 + 4}$$

11. Find dy/dx if
$$y = \frac{(x^3 + 2x)^2(3x - 1)^{1/2}}{(1 - 2x)^2}$$

12. Find dy/dx if
$$y = \frac{\frac{1}{2}x^2}{\ln x}$$

13. Find dy/dx if
$$y = x^{1/2}e^{(x^2+1)}$$

14. The average cost of producing q is given by $AC = \frac{4q}{q+2} + \frac{10,000}{q}$. Find the marginal cost function

15. Find dy/dx if
$$y = \frac{\frac{1}{x} - \frac{7x}{x^2+1}}{\frac{2}{x} - \frac{3x}{x^2+1}}$$

16. If $f(x) = \ln(3x^2 + e^{2x})$ find $f'(x)$.

17. If $f(x) = 4 - x^2 + \frac{1}{2}x^4$ find all the critical points and determine whether they are relative maximums or minimums.

18. Given the function
$$4x^2 + 3xy + y^2 = 8$$
use implicit differentiation find dy/dx when $y = x = 1$.

19. Use implicit differentiation to find $\frac{\partial z}{\partial y}$ when $F(x, y, z) = e^{xy} + 7x^3 + 8z - 19 = 0$

20. If $y = 10\sqrt{x}$, use the method of differentials to find dy when $x = 16$ and $dx = 0.1$.

21. If $f(x, y) = e^{y/x}$ find f_x, f_y, f_{xy}, f_{yx}. Does Young's theorem, which is $f_{xy} = f_{yx}$, hold?.

22. Use method of Lagrange to find the critical points of $f(x, y) = 4x^2 + 2y^2 + 3$ subject to the constraint $x + 2y = 9$.

23. A competitive firm faces a market price of *$21* and has a total cost of
$$C(q) = \frac{1}{12}q^3 - 2.5q^2 + 30q + 100$$
Set up the profit function, find any and all critical points and identify the profit maximizing quantity.

24. Suppose that the output q of a firm depends on the quantities of K and L that it employs as inputs. Its output level is determined by the production function
$$q = 20K + 25L - 5K^2 + 5KL - 5L^2$$

(a) Write down the firm's profit function when the price of q is $p = 1$ and the factor prices of K and L are r and w respectively.

(b) Find the levels of K and L which maximize the firm's profits when $r = 10$ and $w = 6$.

(c) Verify that your solution to (b) satisfies the second order conditions for a maximum.

25. Maximize
$$U(x, y) = 3 \ln x + 2 \ln y$$

Subject to
$$200 \geq 2x + 2y \qquad and \qquad 240 \geq 4x + 2y$$

in addition, the non-negativity constraint $x \geq 0$ and $y \geq 0$.