OPMT 5701

Additional Constrained Optimization Problems

Due: November 24, 2010

Instructions: Below are some additional applications of the lagrange method. Show all your work. These problems are due in lab

1. Maximize
$$u = 4x^2 + 3xy + 6y^2$$
subject to
$$x + y = 56$$
Find the utility maximizing values of x and y.

2. A firm produces two goods, x and y. Due to a government quota, the firm must produce subject to the constraint $x + y = 42$. The firm's cost functions is
$$c(x, y) = 8x^2 - xy + 12y^2$$
Find the cost minimizing values of x and y.

3. A firm wishes to minimize costs subject to an output constraint of $Q = Q_0$. The firm has a production function given by $Q = K^{1/2}L^{1/2}$. If w is the wage rate and r is the rental rate of capital, then the firm's goal is to minimize:
$$TC = wL + rK$$
subject to
$$Q_0 = K^{1/2}L^{1/2}$$
Write down the lagrange function for this problem. Find the cost minimizing values of K and L as functions of w, r and Q_0.

4. Skippy lives on an island where she produces two goods, x *and* y, according the the production possibility frontier $72 = x^2 + y^2$, and she consumes all the goods herself. Her utility function is
$$u = \ln x + 3 \ln y$$
Find her utility maximizing x and y

5. A consumer has the following utility function: $U(x, y) = x(y + 1)$, where x and y are quantities of two consumption goods whose prices are p_x and p_y respectively. The consumer also has a budget of B. Therefore the consumer's maximization problem is
$$x(y + 1) + \lambda(B - p_x x - p_y y)$$
   (a) From the first order conditions find expressions for x^* and y^* expressed as values of $P_x P_y$ and B These are the consumer's demand functions. What kind of good is y? In particular what happens when $p_y > B/2$?

6. This problem could be recast as the following dual problem
$$Minimize \quad p_x x + p_y y \text{ subject to } U_0 = x(y + 1)$$
Find the values of x and y that solve this minimization problem expressed as values of $P_x P_y$ and U_0.