OPMT 5701

Homework #5

More challenging Chain Rule problems

The chain rule, or "function in a function" rule: if $y = f(g(x))$ then $y' = f'(g(x)) \times g'(x)$

It is usually in the form like this

$$y = \overbrace{[g(x)]^n}^{f(x)}$$

where the derivative is

$$y' = \frac{dy}{dx} = \overbrace{n\,[g(x)]^{n-1}}^{f'(x)} \times g'(x)$$

Sometimes the inside function can be a bit more complex. For example

$$y = [h(x)j(x)]^n$$

by breaking it down, we see that

$$y = \overbrace{\left[\underbrace{h(x) \cdot j(x)}_{g(x)}\right]^n}^{f(x)}$$

In this case the inside function, $g(x)$ is two functions, $h(x)$ and $j(x)$. The steps are the same as before except, in this case, we need to use the product rule on $g(x)$ ($g' = h'j + hj'$)

Example: let y be

$$y = \left[(2x+1)(x^{1/2} - 4)\right]^3$$

First, Identify the individual parts:

$$y = \overbrace{\left[\underbrace{(2\overset{h}{x}+1)(x^{1/\overset{j}{2}} - 4)}_{g}\right]^3}^{f}$$

The derivative of f (power-function rule) is

$$f' = 3\left[(2x+1)(x^{1/2}-4)\right]^2$$

and the derivative of g (product rule) is

$$\begin{aligned} g' &= h'j + hj' = (2)(x^{1/2}-4) + (2x+1)\left(\tfrac{1}{2}x^{-1/2}\right) \\ \text{(simplify)} &= 2x^{1/2} - 8 + x^{1/2} + \tfrac{1}{2}x^{-1/2} \\ &= 3x^{1/2} + \tfrac{1}{2}x^{-1/2} - 8 \end{aligned}$$

Now put it all together

$$y' = f' \cdot g' = 3\left[(2x+1)(x^{1/2}-4)\right]^2 \left(x^{1/2} + \tfrac{1}{2}x^{-1/2} - 8\right)$$

Homework Problems

Exercise 1 *find y' if*

$$y = \left[(x^2+x)(2x^3-4)\right]^3$$

Exercise 2 *find y' if*

$$y = \left[(x^{1/3}+2)(5x^{1/5}+7)\right]^2$$

Exercise 3 *find y' if*

$$y = \left[(2x^7+3x)(\tfrac{1}{2}x^2+4)\right]^{1/2}$$

Exercise 4 *find y' if*

$$y = \left[\frac{(3x^2+3)}{(x^3-4)}\right]^2$$

Exercise 5 *find y' if*

$$y = (2x^3+1)^2(x-1)^{1/2}$$

this is tricky! (hint: product, then chain)