OPMT 5701 Marginal Analysis and One Variable Optimization

The following questions involve using the first and second derivatives to sketch curves and verify relative maximums or minimums.

SHORT ANSWER: Show your steps

1) If $f(x) = 2x^3 + 3x^2 - 36x + 1$, determine the intervals on which f is increasing and the intervals on which f is decreasing. 1) ______________

2) Let $f(x) = 3x^5 - 10x^4 + 7x$. Determine the intervals on which f is (a) concave up and (b) concave down. (c) Find the x–values of all inflection points. 2) ______________

3) Let $y = x^3 - 3x^2 - 9x + 10$. 3) ______________
(a) Determine y' and y''.
(b) Determine intervals on which the function is increasing; determine intervals on which the function is decreasing.
(c) Determine the coordinates of all relative maximum and relative minimum points
(d) Determine intervals on which the function is concave up; determine intervals on which the function is concave down;
(e) Determine the coordinates of all inflection points.
(f) With the aid of the information obtained in parts (a)–(e), give a reasonable sketch of the curve.

4) Determine the intervals on which the function is increasing and on which it is decreasing. Also determine the points of relative maxima and relative minima. 4) ______________
$f(x) = (x^2 - 3x + 2)^2$

5) If $y = x^3 + 4x^2 - 3x + 4$, use the second–derivative test to find all values of x for which (a) relative maxima occur (b) relative minima occur. 5) ______________

6) The cost equation for a company is $C(x) = 2x^3 - 15x^2 - 84x + 3100$. Use the second–derivative test, if applicable, to find the relative maxima and the relative minima. 6) ______________

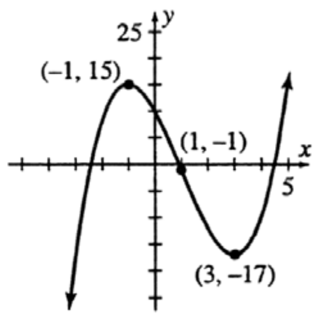
7) The cost equation for a company is $C(x) = 2x^3 - 39x^2 + 180x + 21{,}200$. Use the second–derivative test, if applicable, to find the relative maxima and the relative minima. 7) ______________

8) The demand equation for a monopolist's product is $p = 200 - 0.98q$, where p is the price per unit (in dollars) of producing q units. If the total cost c (in dollars) of producing 8 units is given by $c = 0.02q^2 + 2q + 8000$, find the level of production at which profit is maximized. 8) ______________

9) The demand function for a monopolist's product is $p = 100 - 3q$, where p is the price per unit (in dollars) for q units. If the average cost $\bar{c}$ (in dollars) per unit for q units is $\bar{c} = 4 + \dfrac{100}{q}$, find the output q at which profit is maximized. 9) ______________

10) The demand equation for a monopolist's product is $p = \dfrac{500}{\sqrt{q}}$, where p is the price per unit (in dollars) for q units. If the total cost c (in dollars) of producing q units is given by $c = 5q + 2000$, then the level of production at which profit is maximized is 10) ______________

1) increasing on $(-\infty, -3)$ and $(2, \infty)$; decreasing on $(-3, 2)$
2) (a) $(2, \infty)$ (b) $(-\infty, 0)$ and $(0, 2)$ (c) 2
3) (a) $y' = 3x^2 - 6x - 9$, $y'' = 6x - 6$
   (b) inc. on $(-\infty, -1)$ and $(3, \infty)$; dec. on $(-1, 3)$
   (c) rel. max. at $(-1, 15)$; rel. min. at $(3, -17)$
   (d) conc. up on $(1, \infty)$; conc. down on $(-\infty, 1)$
   (e) $(1, -1)$
   (f)



4) Increasing on the intervals $\left(1, \frac{3}{2}\right)$ and $(2, \infty)$; decreasing on $(-\infty, 1)$ and $\left(\frac{3}{2}, 2\right)$; relative maximum at $x = \frac{3}{2}$; relative minimum at $x = 1, 2$.
5) (a) -3 (b) $\frac{1}{3}$
6) relative maximum when $x = -2$, relative minimum when $x = 7$
7) relative maximum when $x = 3$, relative minimum when $x = 10$
8) 99 units
9) 16
10) 2500 units.