OPMT 5701: Assignment 8: Differential and implicit differentiation

Read Chapters 5.1, 5.2 & appendix 2. Also the web–notes on differentials and Implicit function theorem

Questions 1 to 12 involve using the method of differentials. Sometimes you will need to rearrange terms to get the result necessary.

Questions 13–20 require the implicit function Theorem. Before attempting these problems be sure to review the notes and examples found on the website
(note that the book also covers implicit differentiation in Appendix 2 but with a slightly different approach. Both methods will produce the same result.)

SHORT ANSWER: Show your steps & identify the rules you use.

1) If $y = x \ln x$, then $dy =$ 1) ______________

2) Use differentials and $R = 300x + 50x^2 - x^3$ to approximate the change in revenue R (in dollars) from selling x pounds if the number of pounds increases from 10 to 10.2. 2) ______________

3) Use differentials and $C = 250 + 0.30x$ to approximate the change in the cost C (in dollars) to produce x pounds of candy if the number of pounds of candy increases from 10 pounds to 10.6 pounds. 3) ______________

4) The supply equation for a certain radio is given by $p = 0.8\sqrt{x} + 17$ where p is the price in dollars and x is the number of radios supplied. Use differentials to approximate the price when 620 radios are supplied. (Hint: Use $x = 625$.) 4) ______________

5) The supply equation for a company is $q = 4p^2 + 3$. Find $\frac{dp}{dq}$ from $\frac{dq}{dp}$. 5) ______________

6) The supply equation for a company is $q = \sqrt{300 + p}$. Find $\frac{dp}{dq}$ from $\frac{dq}{dp}$. 6) ______________

7) Use implicit differentiation to find $\frac{dy}{dx}$ explicitly in terms of x and y from $3x^2 + 7xy + y^2 =$ 19. 7) ______________

8) Use implicit differentiation to find $\frac{dy}{dx}$ explicitly in terms of x and y from $xy^2 = e^x + y$. 8) ______________

9) Use implicit differentiation to find $\frac{dy}{dx}$ explicitly in terms of x and y from 9) ______________

$xe^x + (\ln x)y + y^2 = 3$.

10) The volume of a company's sales y (in thousands of dollars) is related to its advertising expenditures x (in thousands of dollars) by the equation $xy - 30x + 14y = 0$. Find $\frac{dy}{dx}$. 10) ______________

11) Suppose that a company can produce 15,000 units when the number of hours of skilled labor y and unskilled labor x satisfy $500 = (x + 1)^{1/4}(y + 9)^{1/5}$. Find $\frac{dy}{dx}$, the rate of change of skilled labor hours with respect to unskilled labor hours. 11) ____________

12) Suppose that a company can produce 12,000 units when the number of hours of skilled labor y and unskilled labor x satisfy $384 = (x + 2)^{3/4}(y + 3)^{1/3}$. Find $\frac{dy}{dx}$, the rate of change of skilled labor hours with respect to unskilled labor hours. 12) ____________

13) If $2x^2 + 3y^2 + 2z^2 = 16$, find $\frac{\partial z}{\partial y}$. 13) ____________

14) If $x^2y + xz + z^2 = 4$, find $\frac{\partial z}{\partial x}$. 14) ____________

15) For $x^2y + xz + z^2 = 4$, evaluate $\frac{\partial z}{\partial x}$ when $x = -1$, $y = 2$, $z = -1$. 15) ____________

16) For $x^2 + xy + yz + z^2 = 6$, the partial derivative $\frac{\partial z}{\partial y}$ evaluated at $x = 1$, $y = 2$, $z = 1$ is 16) ____________

17) For $e^{xy} + 7x^3 + 8z - 18 = 0$, the partial derivative $\frac{\partial z}{\partial y}$ evaluated at $x = -1$, $y = 0$, $z = 3$ is 17) ____________

18) Use implicit partial differentiation to find $\frac{\partial z}{\partial x}$ from $\ln(xyz) = e^y + 79$. 18) ____________

19) Use implicit partial differentiation to find $\frac{\partial z}{\partial y}$ from $e^{xy} + 7x^3 + 8z - 19 = 0$. 19) ____________

20) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate $\frac{\partial z}{\partial y}$ when $x = 1$, $y = 2$, $z = -1$. 20) ____________