

Review your notes on Unconstrained Optimization.

From The text read chapter 5.4 and Appendix 3 (Hessians)

SHORT ANSWER. Solve the following problems. In each case, clearly write out the First Order Conditions and check the Second Order Conditions

- 1) Determine the critical points of $f(x, y) = 3x^2 + 4y^2 - 2x + 8y$ and also determine by the second-derivative test whether each point corresponds to a relative maximum, to a relative minimum, to neither, or whether the test gives no information. 1) _____
- 2) Determine the critical points of $f(x, y) = 4x^2 + 2x - y^2 + 2y$ and also determine by the second-derivative test whether each point corresponds to a relative maximum, to a relative minimum, to neither, or whether the test gives no information. 2) _____
- 3) Determine the critical points of $f(x, y) = 2xy - 3x - y - x^2 - 3y^2$ and also determine by the second-derivative test whether each point corresponds to a relative maximum, to a relative minimum, to neither, or whether the test gives no information. 3) _____
- 4) Determine the critical points of $f(x, y) = x^2 + 2xy + 2y^2 - 4y$ and also determine by the second-derivative test whether each point corresponds to a relative maximum, to a relative minimum, to neither, or whether the test gives no information. 4) _____
- 5) Determine the critical points of $f(x, y) = x^3 + \frac{1}{2}y^2 - 3xy - 4y + 2$ and also determine by the second-derivative test whether each point corresponds to a relative maximum, to a relative minimum, to neither, or whether the test gives no information. 5) _____
- 6) A manufacturer produces products A and B for which the average costs of production are constant at 3 and 5 (dollars per unit), respectively. The quantities q_A, q_B of A and B that can be sold each week are given by the joint-demand functions

$$\begin{aligned} q_A &= 10 - p_A + p_B' \\ q_B &= 12 + p_A - 3p_B'' \end{aligned}$$
 where p_A and p_B are the prices (in dollars per unit) of A and B , respectively. Determine the prices of A and B at which the manufacturer can maximize profit. 6) _____
- 7) Determine the critical points of $f(x, y) = x^2 + xy + y^3 - y$ and also determine by the second-derivative test whether each point corresponds to a relative maximum, to a relative minimum, to neither, or whether the test gives no information. 7) _____
- 8) A monopolist produces two products, A , and B . The joint-cost function is $c = 5q_A + 3q_B + 5000$ where c is the total cost of producing q_A units of A and q_B units of B . the demand functions for these products are given by $p_A = 205 - 2q_A - q_B$ and $p_B = 153 - q_A - q_B$, where p_A and p_B are the prices of A and B , respectively. The number of units of A and the number of units B that should be sold to maximize the monopolist's profit is 8) _____

- 9) A television manufacturing company makes two types of TV's. The cost of producing x units of type A and y units of type B is given by the function $C(x, y) = 100 + x^3 + 64y^3 - 96xy$. How many units of type A and type B televisions should the company produce to minimize its cost? 9) _____
- 10) A television manufacturing company makes two types of TV's. The cost of producing x units of type A and y units of type B is given by the function $C(x, y) = 120 + x^3 + 8y^3 - 24xy$. How many units of type A and type B televisions should the company produce to minimize its cost? 10) _____
- 11) Determine all of the critical points of $f(x, y) = x^3 + 3x^2 - 9x + y^3 - 12y$. Also use the second derivative test to determine, if possible, whether a maximum, minimum or saddle point occurs at each of these critical points. 11) _____
- 12) Determine all of the critical points of $f(x, y) = \frac{1}{3}x^3 + x^2 - 3x + \frac{1}{3}y^3 - 4y$. Also use the second derivative test to determine, if possible, whether a maximum, minimum or saddle point occurs at each of these critical points. 12) _____