Question 5 from Matrix Study Questions

A group of investors has $500,000 to invest in the stocks of three companies. Company A sells for $50 a share and has an expected growth of 13% per year. Company B sells for $20 per share and has an expected growth of 15% per year. Company C sells for $80 a share and has an expected growth of 10% per year.The group decides to try a new investment strategy which entails buying equal amounts of shares in Company B and Company C, and having a goal of 11.4% growth per year. How many shares of each stock should they buy?

First, you need to find the SHARE weighted average return equation, which is

$$\begin{aligned} 0.13(50)A + 0.15(20)B + 0.10(80)C &= 0.114(500000) \\ 6.5A + 3B + 8C &= 57000 \end{aligned}$$

Now you can write this as a system of equations(either 3×3 or 2×2).

First, the three by three (2 by 2 is below)

$$\begin{aligned} 50A + 20B + 80C &= 500000 \\ 6.5A + 3B + 8C &= 57000 \\ B - C &= 0 \end{aligned}$$

Matrix form:

$$\begin{bmatrix} 50 & 20 & 80 \\ 6.5 & 3 & 8 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 500000 \\ 57000 \\ 0 \end{bmatrix}$$

Expand third row

$$\begin{aligned} |A| &= a_{31}|C_{31}| + a_{32}|C_{32}| + a_{33}|C_{33}| \\ &= (0)|C_{31}| + (1)\begin{vmatrix} 50 & 80 \\ 6.5 & 8 \end{vmatrix}(-1)^{3+2} + (-1)\begin{vmatrix} 50 & 20 \\ 6.5 & 3 \end{vmatrix}(-1)^{3+3} \\ &= -(50 \times 8 - 80 \times 6.5) - (50 \times 3 - 20 \times 6.5) = 100 \end{aligned}$$

the cofactor and adjoint are

$$C = \begin{bmatrix} -11 & 6.5 & 6.5 \\ 100 & -50 & -50 \\ -80 & 120.0 & 20.0 \end{bmatrix} \qquad AdjA = \begin{bmatrix} -11 & 100 & -80 \\ 6.5 & -50 & 120.0 \\ 6.5 & -50 & 20.0 \end{bmatrix}$$

The inverse of A is

$$A^{-1} = \begin{bmatrix} -0.11 & 1.0 & -0.8 \\ 0.065 & -0.5 & 1.2 \\ 0.065 & -0.5 & 0.2 \end{bmatrix}$$

Solution

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} -0.11 & 1.0 & -0.8 \\ 0.065 & -0.5 & 1.2 \\ 0.065 & -0.5 & 0.2 \end{bmatrix} \begin{bmatrix} 500000 \\ 57000 \\ 0 \end{bmatrix} = \begin{bmatrix} 2000.0 \\ 4000.0 \\ 4000.0 \end{bmatrix}$$

Two Equation version

$$\begin{aligned} 50A + 100B &= 500000 \\ 6.5A + 11B &= 57000 \end{aligned}$$

Matrix form:

$$\begin{bmatrix} 50 & 100 \\ 6.5 & 11 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 500000 \\ 57000 \end{bmatrix}$$

determinant:$|A| = -100.0$

The Adjoint of A is

$$AdjA = \begin{bmatrix} 11 & -100 \\ -6.5 & 50 \end{bmatrix}$$

the inverse is

$$A^{-1} = \begin{bmatrix} -0.11 & 1.0 \\ 0.065 & -0.5 \end{bmatrix}$$

Now solve the system

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -0.11 & 1.0 \\ 0.065 & -0.5 \end{bmatrix} \begin{bmatrix} 500000 \\ 57000 \end{bmatrix} = \begin{bmatrix} 2000.0 \\ 4000.0 \end{bmatrix}$$

and, since $B = C$, $C = 4000$