Price Disc.

Multiproduct Firm

EXAMPLE:

Let $P_1 = 100 - q_1$ $\quad P_2 = 150 - 2q_2$ $\quad$ MKT. AR functNs

Let $TC = 100 + (q_1 + q_2)^2$

$\pi = \cancel{100}\ P_1 q_1 + P_2 q_2 - 100 - (q_1 + q_2)^2$

$\pi = 100q_1 - q_1^2 + 150q_2 - 2q_2^2 - 100 - (q_1 + q_2)^2$

~~FOCs~~

$\pi_1 = 100 - 2q_1 - 2(q_1 + q_2) = 0 \qquad 100 - 4q_1 - 2q_2 = 0$

$\pi_2 = 150 - 4q_2 - 2(q_1 + q_2) = 0 \qquad 150 - 2q_1 - 6q_2 = 0$

$$q_1 = \frac{\begin{vmatrix} 100 & 2 \\ 150 & 6 \end{vmatrix}}{\begin{vmatrix} 4 & 2 \\ 2 & 6 \end{vmatrix}} = \frac{600 - 300}{20} = 15$$

$$q_2 = \frac{\begin{vmatrix} 4 & 100 \\ 2 & 150 \end{vmatrix}}{20} = \frac{600 - 200}{20} = 20$$

$P_1^* = 85$

$P_2^* = 110$

2nd order Cond.

$$|H| = \begin{vmatrix} -4 & -2 \\ -2 & -6 \end{vmatrix} \qquad H_1 = -4 < 0 \quad H_2 = 20 > 0$$

Therefore, a max

(*) AT HOME, Verify that the Inverse Elasticity Rule Holds here!

Concavity + Convexity

Let $y = f(\bar{x})$ where $\bar{x} = [\bar{x}_1, \ldots, \bar{x}_n]$

and let $\hat{x} = [\hat{x}_1, \ldots, \hat{x}_n]$

such That $\bar{x} \neq \hat{x}$

Definition 1:

$y = f(\bar{x})$ is a concave function

if

$$\underbrace{f(k\bar{x} + (1-k)\hat{x})}_{\text{Point on Dome}} \geq \underbrace{kf(\bar{x}) + (1-k)f(\hat{x})}_{\text{line segment}}$$

Definition 2: $y = f(\bar{x})$ is convex if

$$f(k\bar{x} + (1-k)\hat{x}) \leq kf(\bar{x}) + (1-k)f(\bar{x})$$

for strict Concavity/convexity replace the weak inequalities with strict Inequalities

Concavity and Convexity

If the function $y = f(\bar{X})$ is twice differentiable, then the following holds

Theorem 1:

$y = f(\bar{X})$ is concave/convex if and only the Hessian, $|H|$ is negative/positive semidefinite

Theorem 2:

If the Hessian is negative definite/positive definite for all x, then $y = f(x)$ is concave/convex

Note: Theorem 2 is a sufficient condition for strict concavity/convexity but it is Not a necessary condition

LIMIT-OUTPUT MODEL

Suppose a Monopolist faces the following Demand curve

$$P = a - q$$

a is a constant ≥ 0

His Cost Function is

$$TC = K + cq$$

where K = Setup Costs

cq = Variable Costs

Therefore

$$ATC = \frac{K}{q} + c \quad \{= AFC + AVC\}$$

The Profit function is

$$\pi = Pq - (K + cq)$$

or

$$\pi = (a - q)q - (K + cq)$$

Maximize

$$\frac{\partial \pi}{\partial q} = a - 2q - c = 0 \Rightarrow q = \frac{a-c}{2}$$

Limit OUTPUT (cont.)

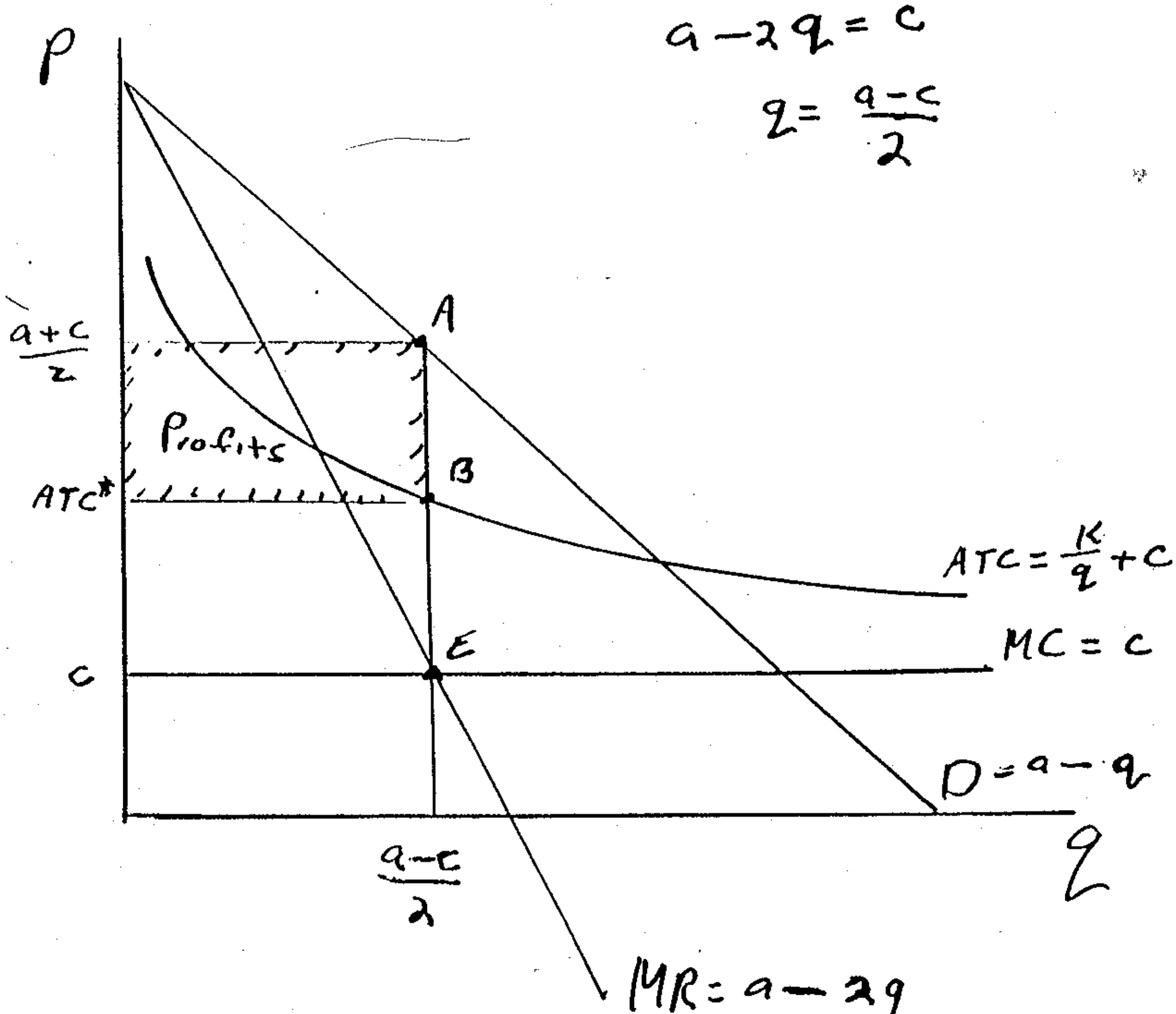
$$P = a - q = a - \left(\frac{a-c}{2}\right) = \frac{a+c}{2}$$

Monopolists
Profit Max.
GRAPHICALLY

set: $MR = MC$

$$a - 2q = c$$

$$q = \frac{a-c}{2}$$



Limit Output Model (cont.)

⇒ Now consider a potential entrant to the monopolist's Market

Assumption: Entrant takes monopolist's output as given

Let: $q_e \equiv$ Entrant's output
$q_m \equiv$ Monopolist's output

If Entrant does enter:
Market price will be:

$$P = a - (q_m + q_e)$$

Entrant's Profits $\quad \pi = pq_e - K - cq_e$

$$\pi_e = (a - q_e - q_m)q_e - K - cq_e$$

$$\frac{\partial \pi_e}{\partial q_e} = a - q_m - 2q_e - c = 0$$

$$q_e = \frac{a - c - q_m}{2}$$

Entrant's output is a function of the monopolist's output.

Limit Output Model (cont.)

Entrants output $q_e = \frac{a-c-q_m}{2}$

Sub into profit function

$$\pi_e = (a - q_m - q_e)q_e - K - cq_e$$

$$\pi_e = (a-q_m)\left(\frac{a-c-q_m}{2}\right) - \left(\frac{a-c-q_m}{2}\right)^2 - K - c\left(\frac{a-c-q_m}{2}\right)$$

Entrant's Profit function is a function of

$a, c, K,$ and q_m

He will enter, if: $\pi_e > 0$

or, if:

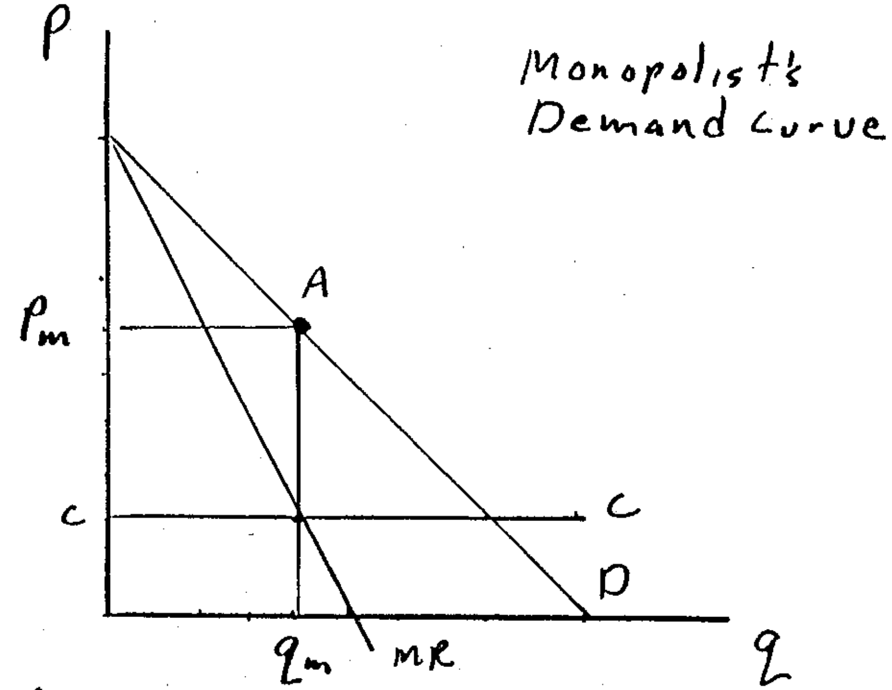
$$(a - q_m - q_e)q_e - cq_e > K$$

Which says: if an entrant's profits (gross) can cover fixed costs (K) then he will enter the market of the monopolist.

Limit output Model (cont.)

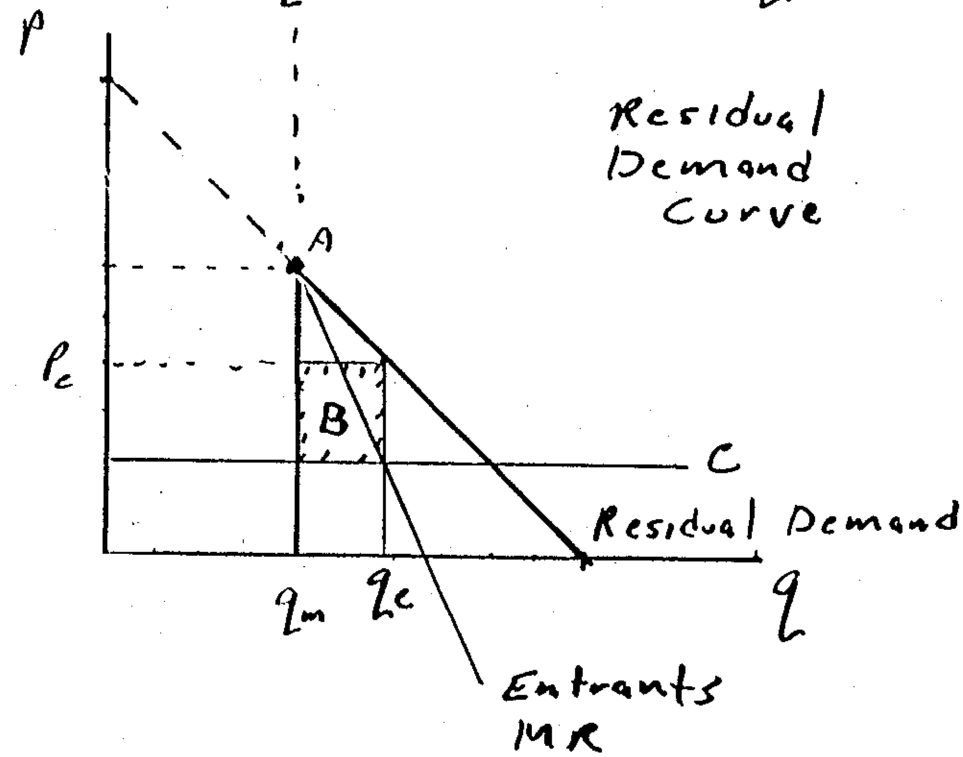
GRAPHICALLY

Entrant takes monopolist's q_m as given



Entrant maximizes profits off the RESIDUAL DEMAND CURVE

B = Entrants profits above variable costs



If $B > K$ then the entrant will enter.

If $B \leq K$ there will be no entry

Limit output (cont.)

The monopolist knows that

$$q_e^* = \frac{a - c - q_m}{2}$$

or generally, $q_e^* = f(q_m)$

⇒ Therefore the monopolist can effect the entrants choice q_e^*

⇒ The monopolist can choose q_m such that when the entrant chooses the optimal q_e^* he will not earn any profits

⇒ Therefore the monopolists maximization problem is:

MAX: $\pi_m = (a - q_m)q_m - K_m - cq_m$

Subject to: $\pi_e = (a - q_m - q_e)q_e - cq_e \leq K_e$

Limit Output (cont.)

sub $q_e = \frac{a-c-q_m}{2}$

into the monopolist's Max problem

Max $aq_m - q_m^2 - cq_m - K$

s.t. $(a-q_m)\left[\frac{a-c-q_m}{2}\right] - \left[\frac{a-c-q_m}{2}\right]^2 - c\left[\frac{a-c-q_m}{2}\right] = K$

⇒ Notice that there is now only <u>one</u> choice Variable, q_m.

⇒ Therefore q_m^* is determined by the constraint.

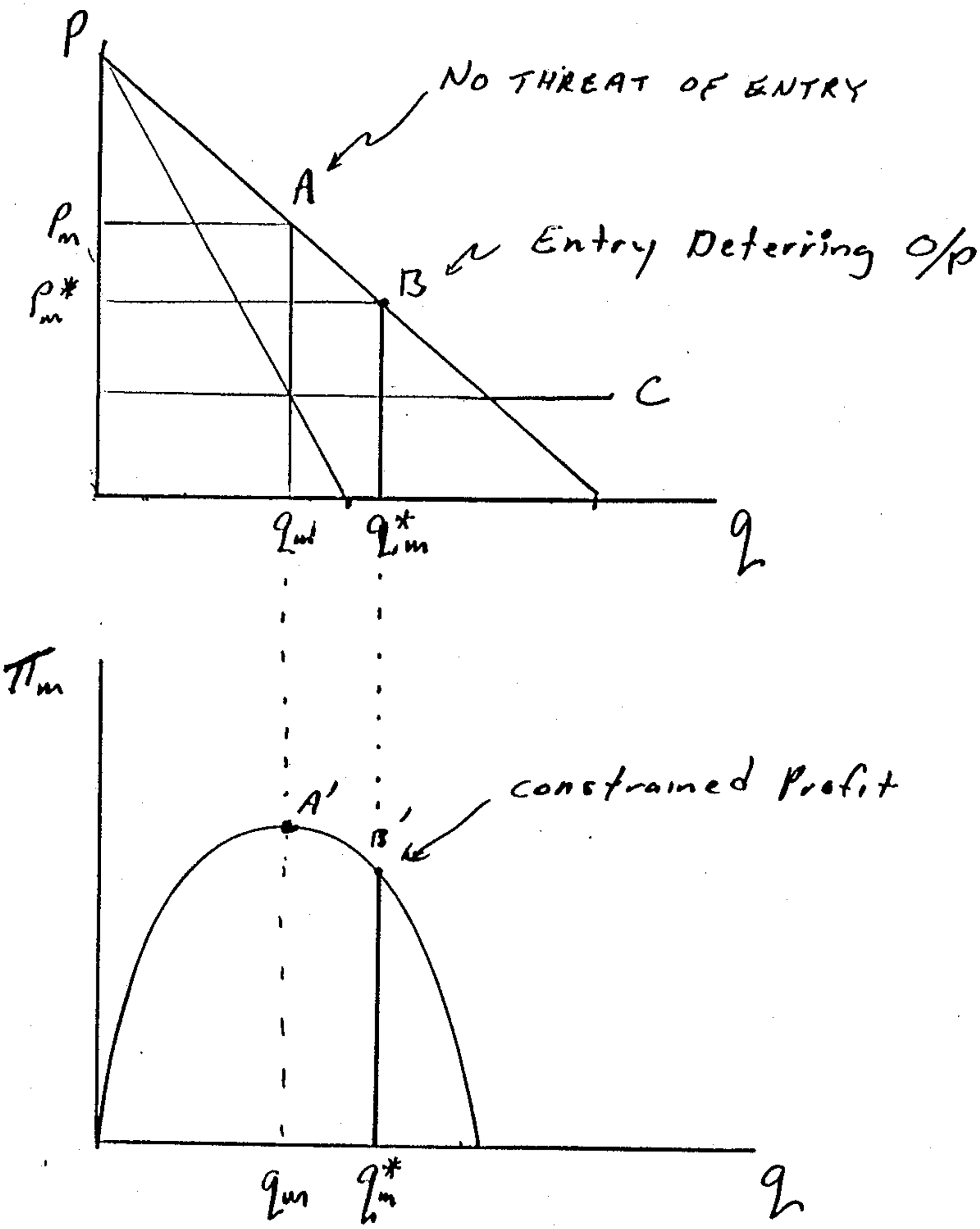
⇒ <u>without</u> differentiating <u>solve</u> the constraint for q_m^*

<u>Answer</u>:

$$q_m^* = a - c - 2\sqrt{K}$$

Limit output (cont.)

GRAPHICALLY



MONOPOLIST PROFIT FUNCTION

COURNOT DUOPOLY
(GAME THEORY)

⇒ SUPPOSE THE MONOPOLIST DECIDES TO ALLOW ENTRY.

THE RESULT: DUOPOLY

⇒ Assumption: Each firm takes the other firms output as exogenous and chooses output to maximize its own profits

⇒ MARKET DEMAND: $P = a - bq$

or $P = a - b(q_1 + q_2)$ $\{q_1 + q_2 = q\}$

where q_i is firm i's output $\{i = 1, 2\}$

⇒ Each firm faces the same cost function

$$TC_i = K + cq_i \qquad \{i = 1, 2\}$$

Each firm's profit function is:

$$\pi_i = Pq_i - cq_i - K$$

Cournot (cont.)

Firm 1:

$$\pi_1 = Pq_1 - cq_1 - K$$

$$\pi_1 = (a - bq_1 - bq_2)q_1 - cq_1 - K$$

Max π_1, treating q_2 as constant

$$\frac{\partial \pi_1}{\partial q_1} = a - bq_2 - 2bq_1 - c = 0$$

$$2bq_1 = a - c - bq_2$$

$$\Rightarrow \quad q_1 = \frac{a-c}{2b} - \frac{q_2}{2} \quad \Rightarrow \text{"Best Response Function"}$$

Best Response function: Tells firm 1 the profit maximizing q_1 for any level of q_2

FOR FIRM 2:

$$\pi_2 = (a - bq_1 - bq_2)q_2 - cq_2 - K$$

Max π_2 (Treating q_1 as constant)

gives $\quad q_2 = \frac{a-c}{2b} - \frac{q_1}{2} \quad$ Firm 2's Best Response Function

COURNOT (cont.)

⇒ The Two "Best Response" functions

(1) $q_1 = \frac{a-c}{2b} - \frac{q_2}{2}$

(2) $q_2 = \frac{a-c}{2b} - \frac{q_1}{2}$

= gives us 2 Equations and Two unknowns.

→ The solution to this system of Equations is the Equilibrium to The "Cournot Duopoly" GAME

⇒ USING CRAMERS RULE:

(1) $q_1^* = \frac{a-c}{3b}$

(2) $q_2^* = \frac{a-c}{3b}$

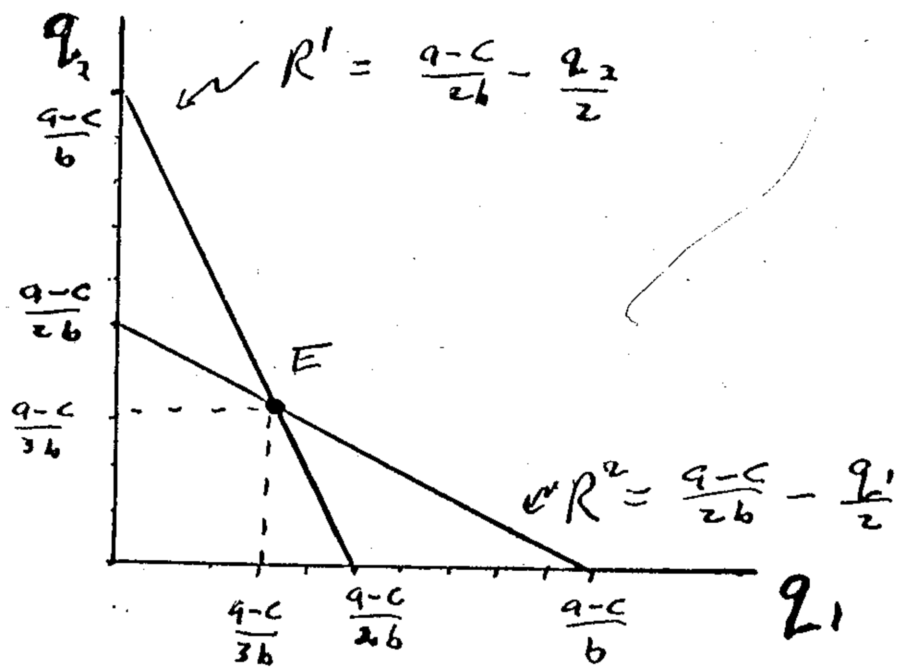
MARKET OUTPUT:

$q_1^* + q_2^* = \frac{2(a-c)}{3b}$

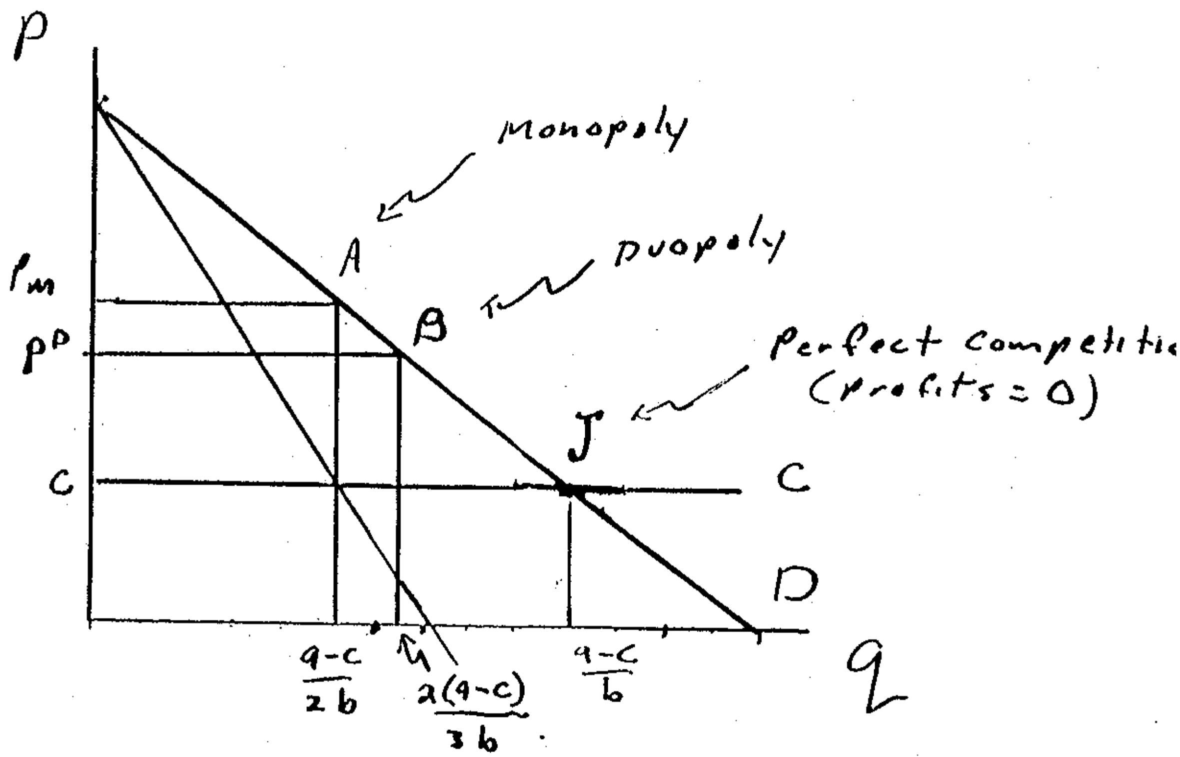
COURNOT (Cont.)

BEST RESPONSE FUNCTIONS

GRAPHICALLY



MARKET SOLUTION



STACKELBERG DUOPOLY

In the Cournot Duopoly, 2 firms picked output simultaneously.

Suppose firm 1 was able to choose output first, knowing how Firm 2's output would vary with firm 1's output

Firm 1's Max Problem

$$\max_{q_1}: (a - bq_1 - bq_2)q_1 - cq_1 - K$$

subject to $q_2 = \frac{a-c}{2b} - \frac{q_1}{2}$ {2's Response function}

Sub in for q_2

$$\max_{q_1}: aq_1 - bq_1^2 - bq_1\left(\frac{a-c}{2b} - \frac{q_1}{2}\right) - cq_1 - K$$

$$\Rightarrow \frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - \left(\frac{a-c}{2}\right) + bq_1 - c = 0$$

$$\Rightarrow q_1^* = \frac{a-c}{2b}$$

STACKELBERG (Cont.)

Firm 2:

$$q_2 = \frac{a-c}{2b} - \frac{q_1}{2}$$

Sub in $q_1 = \frac{a-c}{2b}$

$$q_2^* = \frac{a-c}{2b} - \frac{1}{2}\left(\frac{a-c}{2b}\right) = \frac{a-c}{4b}$$

GRAPHICALLY:

STACKELBERG & COURNOT Equilibrium