

OPMT 5701

Notes on Natural Logarithm and the Exponential e

1. The Number e

$$\text{if } y = e^x \text{ then } \frac{dy}{dx} = e^x$$

$$\text{if } y = e^{f(x)} \text{ then } \frac{dy}{dx} = e^{f(x)} \cdot f'(x)$$

2. Examples

(a)

$$\begin{aligned} y &= e^{3x} \\ \frac{dy}{dx} &= e^{3x}(3) \end{aligned}$$

(b)

$$\begin{aligned} y &= e^{7x^3} \\ \frac{dy}{dx} &= e^{7x^3}(21x^2) \end{aligned}$$

(c)

$$\begin{aligned} y &= e^{rt} \\ \frac{dy}{dt} &= re^{rt} \end{aligned}$$

3. Logarithm (Natural log) $\ln x$

(a) Rules of natural log

<i>If</i>	<i>Then</i>
$y = AB$	$\ln y = \ln(AB) = \ln A + \ln B$
$y = A/B$	$\ln y = \ln A - \ln B$
$y = A^b$	$\ln y = \ln(A^b) = b \ln A$

NOTE: $\ln(A + B) \neq \ln A + \ln B$ EVER!!!

(b) derivatives

<i>IF</i>	<i>THEN</i>
$y = \ln x$	$\frac{dy}{dx} = \frac{1}{x}$
$y = \ln(f(x))$	$\frac{dy}{dx} = \frac{1}{f(x)} \cdot f'(x)$

(c) Examples

i.

$$\begin{aligned} y &= \ln(x^2 - 2x) \\ dy/dx &= \frac{1}{(x^2 - 2x)}(2x - 2) \end{aligned}$$

ii.

$$\begin{aligned} y &= \ln(x^{1/2}) = \frac{1}{2} \ln x \\ dy/dx &= \left(\frac{1}{2}\right) \left(\frac{1}{x}\right) = \frac{1}{2x} \end{aligned}$$