OPMT 5701 THE DERIVATIVE
THE CHAIN RULE

In Problems 1–8, use the chain rule.

***1.** If $y = u^2 - 2u$ and $u = x^2 - x$, find dy/dx.

2. If $y = 2u^3 - 8u$ and $u = 7x - x^3$, find dy/dx.

3. If $y = \frac{1}{w^2}$ and $w = 2 - x$, find dy/dx.

4. If $y = \sqrt[4]{z}$ and $z = x^5 - x^4 + 3$, find dy/dx.

***5.** If $w = u^2$ and $u = \frac{t+1}{t-1}$, find dw/dt when $t = 3$.

6. If $z = u^2 + \sqrt{u} + 9$ and $u = 2s^2 - 1$, find dz/ds when $s = -1$.

7. If $y = 3w^2 - 8w + 4$ and $w = 2x^2 + 1$, find dy/dx when $x = 0$.

8. If $y = 3u^3 - u^2 + 7u - 2$ and $u = 5x - 2$, find dy/dx when $x = 1$.

In Problems 9–52, find y'.

***9.** $y = (3x+2)^6$

10. $y = (x^2 - 4)^4$

11. $y = (3 + 2x^3)^5$

12. $y = (x^2 - x)^3$

13. $y = 2(x^3 - 8x^2 + x)^{100}$

14. $y = \frac{(2x^2+1)^4}{2}$

15. $y = (x^2 - 2)^{-3}$

16. $y = (2x^3 - 8x)^{-12}$

17. $y = 3(2x^2 - 3x - 1)^{-10/3}$

18. $y = 4(7x - x^4)^{-3/2}$

***19.** $y = \sqrt{5x^2 - x}$

20. $y = \sqrt{3x^2 - 7}$

21. $y = \sqrt[4]{2x - 1}$

22. $y = \sqrt[3]{8x^2 - 1}$

23. $y = 2\sqrt[5]{(x^3+1)^2}$

24. $y = 3\sqrt[5]{(x^3-2)^4}$

25. $y = \frac{6}{2x^2 - x + 1}$

26. $y = \frac{3}{x^4 + 2}$

***27.** $y = \frac{1}{(x^2 - 3x)^2}$

28. $y = \frac{1}{(2+x)^4}$

29. $y = \frac{2}{\sqrt{8x - 1}}$

30. $y = \frac{3}{(3x^2 - x)^{2/3}}$

31. $y = \sqrt[3]{7x} + \sqrt[3]{7}x$

32. $y = \sqrt{2x} + \frac{1}{\sqrt{2x}}$

33. $y = x^2(x-4)^5$

34. $y = x(x+4)^4$

35. $y = 4x^2\sqrt{5x+1}$

36. $y = 2x\sqrt{1-x}$

37. $y = (x^2 + 2x - 1)^3(5x)$

38. $y = x^2(x^3 - 1)^4$

***39.** $y = (8x - 1)^3(2x+1)^4$

40. $y = (3x+2)^5(4x-5)^2$

***41.** $y = \left(\frac{x-7}{x+4}\right)^{10}$

42. $y = \left(\frac{2x}{x+2}\right)^4$

43. $y = \sqrt{\frac{x-2}{x+3}}$

44. $y = \sqrt[3]{\frac{8x^2-3}{x^2+2}}$

45. $y = \frac{2x-5}{(x^2+4)^3}$

46. $y = \frac{(4x-2)^4}{3x^2+7}$

47. $y = \frac{(8x-1)^5}{(3x-1)^3}$

48. $y = \sqrt{(x-1)(x+2)^3}$

49. $y = 6(5x^2+2)\sqrt{x^4+5}$

50. $y = 6 + 3x - 4x(7x+1)^2$

51. $y = 8t + \frac{t-1}{t+4} - \left(\frac{8t-7}{4}\right)^2$

52. $y = \frac{(2x^3+6)(7x-5)}{(2x+4)^2}$

In Problems 53 and 54, use the quotient rule and power rule to find y'. Do not simplify your answer.

53. $y = \frac{(2x+1)(3x-5)^2}{(x^2-7)^4}$

54. $y = \frac{\sqrt{x+2}(4x^2-1)^2}{9x-3}$

55. If $y = (5u+6)^3$ and $u = (x^2+1)^4$, find dy/dx when $x = 0$.

56. If $z = 2y^2 - 4y + 5$, $y = 6x - 5$, and $x = 2t$, find dz/dt when $t = 1$.

57. Find the slope of the curve $y = (x^2 - 7x - 8)^3$ at the point $(8, 0)$.

58. Find the slope of the curve $y = \sqrt{x+1}$ at the point $(8, 3)$.

In Problems 59–62, find an equation of the tangent line to t curve at the given point.

59. $y = \sqrt[3]{(x^2-8)^2}$; $(3, 1)$

60. $y = (3x+1)^2$; $(-1, 4)$

61. $y = \frac{\sqrt{7x+2}}{x+1}$; $\left(1, \frac{3}{2}\right)$

62. $y = \frac{-3}{(3x^2+1)^3}$; $(0, -3)$

In Problems 63 and 64, determine the percentage rate of change of y with respect to x for the given value of x.

63. $y = (x^2+9)^3$; $x = 4$

64. $y = \frac{1}{(x^2-1)^3}$; $x = 2$

In Problems 65–68, q is the total number of units produced per day by m employees of a manufacturer, and p is the pric per unit at which the q units are sold. In each case, find the marginal-revenue product for the given value of m.

65. $q = 2m$, $p = -0.5q + 20$; $m = 5$

66. $q = (200m - m^2)/20$, $p = -0.1q + 70$; $m = 40$

67. $q = 10m^2/\sqrt{m^2+9}$, $p = 525/(q+3)$; $m = 4$

68. $q = 100m/\sqrt{m^2+19}$, $p = 4500/(q+10)$; $m = 9$

69. Demand Equation Suppose $p = 100 - \sqrt{q^2+20}$ is a demand equation for a manufacturer's product.

(a) Find the rate of change of p with respect to q.

(b) Find the relative rate of change of p with respect to q.

(c) Find the marginal-revenue function.

70. Marginal-Revenue Product If $p = k/q$, where k is a constant, is the demand equation for a manufacturer's product and $q = f(m)$ defines a function that gives the total number of units produced per day by m employees, show that the marginal-revenue product is always zero.

71. Cost Function The cost c of producing q units of a product is given by

$$c = 5500 + 12q + 0.2q^2$$

If the price per unit p is given by the equation

$$q = 900 - 1.5p$$

use the chain rule to find the rate of change of cost with respect to price per unit when $p = 85$.