OPMT 5701 THE DERIVATIVE
PRODUCT AND QUOTIENT RULES

In Problems 1–48, differentiate the functions.

*1. $f(x) = (4x+1)(6x+3)$

2. $f(x) = (3x-1)(7x+2)$

3. $s(t) = (5-3t)(t^3-2t^2)$

4. $Q(x) = (5-2x)(x^2+1)$

5. $f(r) = (3r^2-4)(r^2-5r+1)$

6. $C(I) = (2I^2-3)(3I^2-4I+1)$

7. $f(x) = x^2(2x^2-5)$

8. $f(x) = 3x^3(x^2-2x+2)$

9. $y = (x^2+3x-2)(2x^2-x-3)$

10. $\phi(x) = (3-5x+2x^2)(2+x-4x^2)$

11. $f(w) = (8w^2+2w-3)(5w^3+2)$

12. $f(x) = (3x-x^2)(3-x-x^2)$

13. $y = (x^2-1)(3x^3-6x+5) - 4(4x^2+2x+1)$

14. $h(x) = 4(x^5-3) + 3(8x^2-5)(2x+2)$

*15. $F(p) = \frac{3}{2}(5\sqrt{p}-2)(3p-1)$

16. $g(x) = (\sqrt{x}-3x+1)(\sqrt[4]{x}-2\sqrt{x})$

*17. $y = 7 \cdot \frac{2}{3}$

18. $y = (x-1)(x-2)(x-3)$

*19. $y = (2x-1)(3x+4)(x+7)$

20. $y = \dfrac{2x-3}{4x+1}$

*21. $f(x) = \dfrac{5x}{x-1}$

22. $H(x) = \dfrac{-5x}{5-x}$

23. $f(x) = \dfrac{3}{2x^6}$

24. $f(x) = \dfrac{5(x^2-2)}{7}$

25. $y = \dfrac{x+2}{x-1}$

26. $h(w) = \dfrac{3w^2+5w-1}{w-3}$

*27. $h(z) = \dfrac{6-2z}{z^2-4}$

28. $z = \dfrac{3x^2-x+1}{x^2+2x+5}$

29. $y = \dfrac{8x^2-2x+1}{x^2-5x}$

30. $f(x) = \dfrac{x^3-x^2+1}{x^2+1}$

31. $y = \dfrac{x^2-4x+3}{2x^2-3x+2}$

32. $F(z) = \dfrac{z^4+4}{3z}$

33. $g(x) = \dfrac{1}{x^{100}+7}$

34. $y = \dfrac{-9}{2x^5}$

35. $u(v) = \dfrac{v^5-8}{v}$

36. $y = \dfrac{x-5}{8\sqrt{x}}$

37. $y = \dfrac{3x^2-x-1}{\sqrt[3]{x}}$

38. $y = \dfrac{x^{0.3}-2}{2x^{2.1}+1}$

39. $y = 7 - \dfrac{4}{x-8} + \dfrac{2x}{3x+1}$

40. $q(x) = 3x^2 + \dfrac{3x-2}{x+2} - \dfrac{4}{x^2}$

41. $y = \dfrac{x-5}{(x+2)(x-4)}$

42. $y = \dfrac{(9x-1)(3x+2)}{4-5x}$

43. $s(t) = \dfrac{t^2+3t}{(t^2-1)(t^3+7)}$

44. $f(s) = \dfrac{17}{s(5s^2-10s+4)}$

*45. $y = 3x - \dfrac{\frac{2}{x} - \frac{3}{x-1}}{x-2}$

46. $y = 3 - 12x^3 + \dfrac{1 - \frac{5}{x^2+2}}{x^2+5}$

47. $f(x) = \dfrac{a-x}{a+x}$, where a is a constant

48. $f(x) = \dfrac{x^{-1}+a^{-1}}{x^{-1}-a^{-1}}$, where a is a constant, $a \neq 0$

*49. Find the slope of the curve $y = (4x^2+2x-5)(x^3+7x+4)$ at $(-1, 12)$.

50. Find the slope of the curve $y = \dfrac{x^3}{x^4+1}$ at $(-1, -\frac{1}{2})$.

In Problems 51–54, find an equation of the tangent line to the curve at the given point.

51. $y = \dfrac{6}{x-1}$; $(3, 3)$

52. $y = \dfrac{2x+3}{x^2}$; $(1, 5)$

53. $y = (2x+3)[2(x^4-5x^2+4)]$; $(0, 24)$

54. $y = \dfrac{x+1}{x^2(x-4)}$; $(2, -\frac{3}{8})$

In Problems 55 and 56, determine the relative rate of change of y with respect to x for the given value of x.

55. $y = \dfrac{x}{2x-6}$; $x = 1$

56. $y = \dfrac{1-x}{1+x}$; $x = 5$

57. **Motion** The position function for an object moving in a straight line is

$$s = \frac{2}{t^3+1}$$

where t is in seconds and s is in meters. Find the position and velocity of the object at $t = 1$.

58. **Motion** The position function for an object moving in a straight-line path is

$$s = \frac{t+3}{t^2+7}$$

where t is in seconds and s is in meters. Find the positive value(s) of t for which the velocity of the object is 0.

In Problems 59–62, each equation represents a demand function for a certain product, where p denotes the price per unit for q units. Find the marginal-revenue function in each case. Recall that revenue = pq.

*59. $p = 25 - 0.02q$

60. $p = 500/q$