OPMT 5701 multivariable optimization

In Problems 1–6, find the critical points of the functions.

1. $f(x, y) = x^2 + y^2 - 5x + 4y + xy.$

2. $f(x, y) = x^2 + 4y^2 - 6x + 16y.$

3. $f(x, y) = 2x^3 + y^3 - 3x^2 + 1.5y^2 - 12x - 90y.$

4. $f(x, y) = xy - \dfrac{1}{x} - \dfrac{1}{y}.$

5. $f(x, y, z) = 2x^2 + xy + y^2 + 100 - z(x + y - 200).$

6. $f(x, y, z, w) = x^2 + y^2 + z^2 - w(x - y + 2z - 6).$

In Problems 7–20, find the critical points of the functions. For each critical point, determine, by the second-derivative test, whether it corresponds to a relative maximum, to a relative minimum, or to neither, or whether the test gives no information.

7. $f(x, y) = x^2 + 3y^2 + 4x - 9y + 3.$

8. $f(x, y) = -2x^2 + 8x - 3y^2 + 24y + 7.$

9. $f(x, y) = y - y^2 - 3x - 6x^2.$

10. $f(x, y) = x^2 + y^2 + xy - 9x + 1.$

11. $f(x, y) = x^3 - 3xy + y^2 + y - 5.$

12. $f(x, y) = \dfrac{x^3}{3} + y^2 - 2x + 2y - 2xy.$

13. $f(x, y) = \dfrac{1}{3}(x^3 + 8y^3) - 2(x^2 + y^2) + 1.$

14. $f(x, y) = x^2 + y^2 - xy + x^3.$

15. $f(l, k) = 2lk - l^2 + 264k - 10l - 2k^2.$

16. $f(l, k) = l^3 + k^3 - 3lk.$

17. $f(p, q) = pq - \dfrac{1}{p} - \dfrac{1}{q}.$

18. $f(x, y) = (x - 3)(y - 3)(x + y - 3).$

19. $f(x, y) = (y^2 - 4)(e^x - 1).$

20. $f(x, y) = \ln(xy) + 2x^2 - xy - 6x.$

In Problems 21–35, unless otherwise indicated, the variables p_A and p_B denote selling prices of products A and B, respectively. Similarly, q_A and q_B denote quantities of A and B that are produced and sold during some time period. In all cases, the variables employed will be assumed to be units of output, input, money, etc.

21. Maximizing Output Suppose

$$P = f(l, k) = 1.08l^2 - 0.03l^3 + 1.68k^2 - 0.08k^3$$

is a production function for a firm. Find the quantities of inputs l and k so as to maximize output P.

22. Maximizing Output In a certain automated manufacturing process, machines M and N are utilized for m and n hours, respectively. If daily output Q is a function of m and n, namely,

$$Q = 4.5m + 5n - 0.5m^2 - n^2 - 0.25mn,$$

find the values of m and n that maximize Q.

23. Profit A candy company produces two varieties of candy, A and B, for which the constant average costs of production are 60 and 70 (cents per lb), respectively. The demand functions for A and B are given by

$$q_A = 5(p_B - p_A) \quad \text{and} \quad q_B = 500 + 5(p_A - 2p_B).$$

Find the selling prices p_A and p_B that maximize the company's profit.

24. Profit Repeat Problem 23 if the constant costs of production of A and B are a and b (cents per lb), respectively.

25. Price Discrimination Suppose a monopolist is practicing price discrimination in the sale of a product by charging different prices in two separate markets. In market A the demand function is

$$p_A = 100 - q_A,$$

and in B it is

$$p_B = 84 - q_B,$$

where q_A and q_B are the quantities sold per week in A and B, and p_A and p_B are the respective prices per unit. If the monopolist's cost function is

$$c = 600 + 4(q_A + q_B),$$

how much should be sold in each market to maximize profit? What selling prices give this maximum profit? Find the maximum profit.

26. Profit A monopolist sells two competitive products, A and B, for which the demand functions are

$$q_A = 1 - 2p_A + 4p_B \quad \text{and} \quad q_B = 11 + 2p_A - 6p_B.$$

If the constant average cost of producing a unit of A is 4 and a unit of B is 1, how many units of A and B should be sold to maximize the monopolist's profit?

27. Profit For products A and B, the joint-cost function for a manufacturer is

$$c = 1.5q_A^2 + 4.5q_B^2,$$

and the demand functions are $p_A = 36 - q_A^2$ and $p_B = 30 - q_B^2$. Find the level of production that maximizes profit.

28. Profit For a monopolist's products A and B, the joint-cost function is $c = (q_A + q_B)^2$, and the demand functions are $q_A = 26 - p_A$ and $q_B = 10 - 0.25p_B$. Find the values of p_A and p_B that maximize profit. What are the quantities of A and B that correspond to these prices? What is the total profit?

29. Cost An open-top rectangular box is to have a volume of 6 ft^3. The cost per square foot of materials is \$3 for the bottom, \$1 for the front and back, and \$0.50 for the other two sides. Find the dimensions of the box so that the cost of materials is minimized. (See Fig. 19.15.)

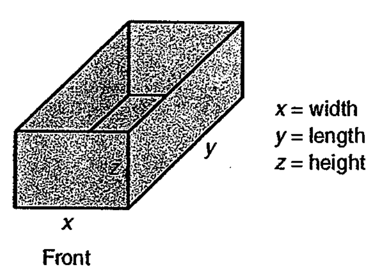

FIGURE 19.15 Diagram for Problem 29.

answers to odd # Optimization

EXERCISE 19.7 (page 913)

1. $\left(\frac{14}{3}, -\frac{13}{3}\right)$. **3.** $(2, 5), (2, -6), (-1, 5), (-1, -6)$.

5. $(50, 150, 350)$. **7.** $\left(-2, \frac{3}{2}\right)$, rel. min.

9. $\left(-\frac{1}{4}, \frac{1}{2}\right)$, rel. max.

11. $(1, 1)$, rel. min; $\left(\frac{1}{2}, \frac{1}{4}\right)$, neither.

13. $(0, 0)$, rel. max.; $\left(4, \frac{1}{2}\right)$, rel. min.; $\left(0, \frac{1}{2}\right)$, $(4, 0)$, neither.

15. $(122, 127)$, rel. max. **17.** $(-1, -1)$, rel. min.

19. $(0, -2), (0, 2)$, neither. **21.** $l = 24, k = 14$.

23. $p_A = 80, p_B = 85$.

25. $q_A = 48, q_B = 40, p_A = 52, p_B = 44$, profit $= 3304$.

27. $q_A = 3, q_B = 2$. **29.** 1 ft by 2 ft by 3 ft.

31. $\left(\frac{105}{37}, \frac{28}{37}\right)$, rel. min. **33.** $a = -8, b = -12, d = 33$.

35. a. 2 units of A and 3 units B;
b. Selling price for A is 30 and selling price for B is 19. Relative maximum profit is 25.

37. a. $P = 5T(1 - e^{-x}) - 20x - 0.1T^2$;
c. Relative maximum at $(20, \ln 5)$; no relative extremum at $\left(5, \ln \frac{5}{4}\right)$.