OPMT 5701 Study Questions
Partial Derivatives

Exercise 19.4

In Problems 1–11, find the indicated partial derivatives by the method of implicit partial differentiation.

1. $x^2 + y^2 + z^2 = 9$; $\partial z/\partial x$.

2. $z^2 - 5x^2 + y^2 = 0$; $\partial z/\partial x$.

3. $2z^3 - x^2 - 4y^2 = 0$; $\partial z/\partial y$.

4. $3x^2 + y^2 + 2z^3 = 9$; $\partial z/\partial y$.

5. $x^2 - 2y - z^2 + x^2yz^2 = 20$; $\partial z/\partial x$.

6. $z^3 - xz - y = 0$; $\partial z/\partial x$.

7. $e^x + e^y + e^z = 10$; $\partial z/\partial y$.

8. $xyz + 2y^2x - z^3 = 0$; $\partial z/\partial x$.

9. $\ln(z) + 9z - xy = 1$; $\partial z/\partial x$.

10. $\ln x + \ln y - \ln z = e^y$; $\partial z/\partial x$.

11. $(z^2 + 6xy)\sqrt{x^3 + 5} = 2$; $\partial z/\partial y$.

In Problems 12–20, evaluate the indicated partial derivatives for the given values of the variables.

12. $xz + xyz - 5 = 0$; $\partial z/\partial x$, $x = 1$, $y = 4$, $z = 1$.

13. $xz^2 + yz - 12 = 0$; $\partial z/\partial x$, $x = 2$, $y = -2$, $z = 3$.

14. $e^{zx} = xyz$; $\partial z/\partial y$, $x = 1$, $y = -e^{-1}$, $z = -1$.

15. $e^{yz} = -xyz$; $\partial z/\partial x$, $x = -e^2/2$, $y = 1$, $z = 2$.

16. $\sqrt{xz + y^2} - xy = 0$; $\partial z/\partial y$, $x = 2$, $y = 2$, $z = 6$.

17. $\ln z = 4x + y$; $\partial z/\partial x$, $x = 5$, $y = -20$, $z = 1$.

18. $\dfrac{rs}{s^2 + t^2} = t$; $\partial r/\partial t$, $r = 0$, $s = 1$, $t = 0$.

19. $\dfrac{s^2 + t^2}{rs} = 10$; $\partial t/\partial r$, $r = 1$, $s = 2$, $t = 4$.

20. $\ln(x + z) + xyz = x^2e^{y+z}$; $\partial z/\partial x$, $x = 0$, $y = 1$, $z = 1$.

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21. Joint-Cost Function A joint-cost function is defined implicitly by the equation

$$c + \sqrt{c} = 12 + q_A\sqrt{9 + q_B^2},$$

where c denotes the total cost (in dollars) for producing q_A units of product A and q_B units of product B.

a. If $q_A = 6$ and $q_B = 4$, find the corresponding value of c.

b. Determine the marginal costs with respect to q_A and q_B when $q_A = 6$ and $q_B = 4$.

EXERCISE 19.4 (page 898)

1. $-\dfrac{x}{z}$. **3.** $\dfrac{4y}{3z^2}$. **5.** $\dfrac{x(yz^2 + 1)}{z(1 - x^2y)}$. **7.** $-e^{y-z}$.

9. $\dfrac{yz}{9 + z}$. **11.** $-\dfrac{3x}{z}$. **13.** $-\dfrac{9}{10}$. **15.** $-\dfrac{4}{e^2}$.

17. 4. **19.** $\dfrac{5}{2}$.

21. a. 36; **b.** With respect to q_A, $\dfrac{60}{13}$; with respect to q_B, $\dfrac{288}{65}$.

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Exercise 19.5

In Problems 1–10, find the indicated partial derivatives.

1. $f(x, y) = 4x^2y; \quad f_x(x, y), f_{xy}(x, y)$.

2. $f(x, y) = 4x^3 + 5x^2y^3 - 3y; \quad f_x(x, y), f_{xx}(x, y)$.

3. $f(x, y) = 7x^2 + 3y; \quad f_y(x, y), f_{yy}(x, y), f_{yyx}(x, y)$.

4. $f(x, y) = (x^2 + xy + y^2)(x^2 + xy + 1)$; $f_x(x, y), f_{xy}(x, y)$.

5. $f(x, y) = 9e^{2xy}; \quad f_y(x, y), f_{yx}(x, y), f_{yxy}(x, y)$.

6. $f(x, y) = \ln(x^2 + y^2) + 2; \quad f_x(x, y), f_{xx}(x, y), f_{xy}(x, y)$.

7. $f(x, y) = (x + y)^2(xy); \quad f_x(x, y), f_y(x, y), f_{xx}(x, y)$, $f_{yy}(x, y)$.

8. $f(x, y, z) = xy^2z^3; \quad f_x(x, y, z), f_{xz}(x, y, z), f_{xy}(x, y, z)$.

9. $z = \sqrt{x^2 + y^2}; \quad \dfrac{\partial z}{\partial x}, \dfrac{\partial^2 z}{\partial x^2}$.

10. $z = \dfrac{\ln(x^2 + 5)}{y}; \quad \dfrac{\partial z}{\partial x}, \dfrac{\partial^2 z}{\partial y\, \partial x}$.

In Problems 11–16, find the indicated value.

11. If $f(x, y, z) = 7$, find $f_{yxx}(4, 3, -2)$.

12. If $f(x, y, z) = z^2(3x^2 - 4xy^3)$, find $f_{xyz}(1, 2, 3)$.

13. If $f(l, k) = 5l^3k^6 - lk^7$, find $f_{kkl}(8, 1)$.

14. If $f(x, y) = 2x^2y + xy^2 - x^2y^2$, find $f_{xxy}(5, 1)$.

15. If $f(x, y) = y^2e^x + \ln(xy)$, find $f_{xyy}(1, 1)$.

16. If $f(x, y) = x^3 - 6xy^2 + x^2 - y^3$, find $f_{xy}(1, -1)$.

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17. Cost Function Suppose the cost c of producing q_A units of product A and q_B units of product B is given by

$$c = (3q_A^2 + q_B^3 + 4)^{1/3},$$

and the coupled demand functions for the products are given by

$$q_A = 10 - p_A + p_B^2$$

and

$$q_B = 20 + p_A - 11p_B.$$

Find the value of

$$\frac{\partial^2 c}{\partial q_A\, \partial q_B}$$

when $p_A = 25$ and $p_B = 4$.

18. For $f(x, y) = x^4y^4 + 3x^3y^2 - 7x + 4$, show that

$$f_{xyx}(x, y) = f_{xxy}(x, y).$$

19. For $f(x, y) = 8x^3 + 2x^2y^2 + 5y^4$, show that

$$f_{xy}(x, y) = f_{yx}(x, y).$$

20. For $f(x, y) = xe^{y/x}$, show that

$$xf_{xx}(x, y) + yf_{xy}(x, y) = 0.$$

21. For $z = \ln(x^2 + y^2)$, show that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$.

[19]**22.** If $2z^2 - x^2 - 4y^2 = 0$, find $\dfrac{\partial^2 z}{\partial x^2}$.

[19]**23.** If $z^2 - 3x^2 + y^2 = 0$, find $\dfrac{\partial^2 z}{\partial y^2}$.

[19]**24.** If $2z^2 = x^2 + 2xy + xz$, find $\dfrac{\partial^2 z}{\partial x\, \partial y}$.

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[19]Omit if Sec. 19.4 was not covered.

EXERCISE 19.5 (page 901)

1. $8xy; 8x$. **3.** $3; 0; 0$.
5. $18xe^{2xy}; 18e^{2xy}(2xy + 1); 72x(1 + xy)e^{2xy}$.
7. $3x^2y + 4xy^2 + y^3; 3xy^2 + 4x^2y + x^3; 6xy + 4y^2$; $6xy + 4x^2$. **9.** $x(x^2 + y^2)^{-1/2}; y^2(x^2 + y^2)^{-3/2}$.
11. 0. **13.** 28,758. **15.** $2e$.
17. $-\dfrac{1}{8}$. **23.** $-\dfrac{y^2 + z^2}{z^3} = -\dfrac{3x^2}{z^3}$.