OPMT 5701 Study Questions
Partial Derivatives

Lagrange Method

Exercise 19.8

In Problems 1–12 find, by the method of Lagrange multipliers, the critical points of the functions, subject to the given constraints.

1. $f(x, y) = x^2 + 4y^2 + 6;\ 2x - 8y = 20.$
2. $f(x, y) = -2x^2 + 5y^2 + 7;\ 3x - 2y = 7.$
3. $f(x, y, z) = x^2 + y^2 + z^2;\ 2x + y - z = 9.$
4. $f(x, y, z) = x + y + z;\ xyz = 27.$
5. $f(x, y, z) = x^2 + xy + 2y^2 + z^2;\ x - 3y - 4z = 16.$
6. $f(x, y, z) = xyz^2;\ x - y + z = 20\ (xyz^2 \neq 0).$
7. $f(x, y, z) = xyz;\ x + 2y + 3z = 18\ (xyz \neq 0).$
8. $f(x, y, z) = x^2 + y^2 + z^2;\ x + y + z = 3.$
9. $f(x, y, z) = x^2 + 2y - z^2;\ 2x - y = 0,\ y + z = 0.$
10. $f(x, y, z) = x^2 + y^2 + z^2;\ x + y + z = 4,$ $x - y + z = 4$
11. $f(x, y, z) = xyz;\ x + y + z = 12,$ $x + y - z = 0 (xyz \neq 0).$
12. $f(x, y, z, w) = 2x^2 + 2y^2 + 3z^2 - 4w^2;$ $4x - 8y + 6z + 16w = 6.$

13. **Production Allocation** To fill an order for 100 units of its product, a firm wishes to distribute production between its two plants, plant 1 and plant 2. The total-cost function is given by

$$c = f(q_1, q_2) = 0.1q_1^2 + 7q_1 + 15q_2 + 1000,$$

where q_1 and q_2 are the numbers of units produced at plants 1 and 2, respectively. How should the output be distributed in order to minimize costs? (You may assume that the critical point obtained does correspond to the minimum cost.)

14. **Production Allocation** Repeat Problem 13 if the cost function is

$$c = 3q_1^2 + q_1q_2 + 2q_2^2$$

and a total of 200 units are to be produced.

15. **Maximizing Output** The production function for a firm is

$$f(l, k) = 12l + 20k - l^2 - 2k^2.$$

The cost to the firm of l and k is 4 and 8 per unit, respectively. If the firm wants the total cost of input to be 88, find the greatest output possible, subject to this budget constraint. (You may assume that the critical point obtained does correspond to the maximum output.)

16. **Maximizing Output** Repeat Problem 15, given that

$$f(l, k) = 60l + 30k - 2l^2 - 3k^2$$

and the budget constraint is $2l + 3k = 30$.

17. **Advertising Budget** A computer company has a monthly advertising budget of \$60,000. Its marketing department estimates that if x dollars are spent each month on advertising in newspapers and y dollars per month on television advertising, then the monthly sales will be given by $S = 90x^{1/4}y^{3/4}$ dollars. If the profit is 10% of sales, less the advertising cost, determine how to allocate the advertising budget in order to maximize the monthly profit. (You may assume that the critical point obtained does correspond to the maximum profit.)

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1. $(2, -2)$. **3.** $\left(3, \frac{3}{2}, -\frac{3}{2}\right)$. **5.** $\left(\frac{4}{3}, -\frac{4}{3}, -\frac{8}{3}\right)$.
7. $(6, 3, 2)$. **9.** $\left(\frac{2}{3}, \frac{4}{3}, -\frac{4}{3}\right)$. **11.** $(3, 3, 6)$.
13. Plant 1, 40 units; plant 2, 60 units.
15. 74 units (when $l = 8, k = 7$).
17. \$15,000 on newspaper advertising and \$45,000 on TV advertising.
19. $x = 5, y = 15, z = 5$.
21. $x = 12, y = 8$. **23.** $x = 10, y = 20, z = 5$.