British Columbia Institute of Technology

Practice on Game Theory

Problem 1. SET UP COSTS AND MARKET STRUCTURE.
Market demand is $P = 100 - Q$, where Q is the total quantity sold in the market. Suppose that in order to enter the industry a firm has to incur set up cost and this cost will translate into fixed cost of FC =\$400 per year. Once the firm is in the market it will have variable costs $VC = 20q$, therefore total costs are $TC = 20q + 400$. Currently there is an incumbent firm in the market, let's denote firm I. The firm is aware that more firms can potentially enter the market is considering a strategy to deter entry. At first let's consider what the incumbent firm can do to prevent more firms from entering the market and then use game theory to analyze whether this strategy will really work in a sequential game.

Entry Deterrence Firm I knows that the new firm, let's call it entrant and dentote E will enter the market if it can make profits sufficient to cover its fixed costs: $(P - AVC)q_E > 400$. Firm I can strategically lower π_E by increasing output q_I in order to lower the market price up to the point when the entrant cannot cover its fixed costs. Let's find the output of the incumbent that will prevent entry. The logic is as follows: incumbent chooses q_I, which entrant observes and takes as given; the entrant calculates the maximum profits he can earn in the market and if the profits are greater then zero, enters the market.

- ***E chooses*** q_E *to max* π_E. At this point incumbent's output is already chosen, let's denote it $\bar{q}_I$. Write down entrant's profits as a function of $\bar{q}_I$ and q_E:

  $\pi_E =$

  to maximize profits:

  $\frac{\partial \pi_E}{\partial q_E} =$

  Solve for $q_E^* =$

  This tells you what will be entrant's optimal output in response to any level of output chosen by the incumbent. Notice that this is E's best response to the output of firm I.

- ***Decision to enter*** You need to calculate entrant's profits, for which you have to calculate the market price. Use the demand function, $Q = \bar{q}_I + q_E$, plug q_E* to find 'after entry' market price for any output chosen by the incumbent firm (will be a function of $\bar{q}_I$):

  $P_E =$

  Entry is profitable if $(P_E - AVC)q_E > 400$. The LHS represents the profits net of the fixed cost: how much the firm gets per unit, minus the variable costs of producing that unit times output. Substitute for P_E, AVC, and q_E^* for q_E to find the condition for entry:

  > 400

then if $\bar{q}_I <$, it is profitable for one more firm to enter the market. If $\bar{q}_I \geq$ entry is not profitable.

In conclusion, if the incumbent sets $\bar{q}_I =$ and charges price $\bar{P} =$ he can prevent other firms from entering the market. The incumbent's profits in case it deters entry

$\pi_I = (\bar{P} - 20)\bar{q}_I - 400 =$

Does it make sense for the incumbent to expand output and lower the price? In this $q = 40$ is the output that a monopoly would produce to max profits in any case[1], but in general, would producing smaller output and charging higher price generate more profits? *(Think about what will happen to market* ***price*** *in this setting: unless the incumbent maintains* $\bar{q}_I$ *new firms will keep entering the market until... ? What happens to I's profits in this case?)*

Will the story end with one firm in the market? Let's now think about this game from the entrant's perspective. Suppose that firm I chooses the high output to deter entry, but firm E still enters the market. Is it in the best interest of the firm I to maintain the high output? Calculate firms' and market output and market price if the two firms play Cournot duopoly game. Do you think that a third firm will enter the market? *(Hint: compare total output in Cournot game to the output that is sufficient to deter entry).*

Problem 2. SPATIAL LOCATION GAME.

Suppose that there are two sellers with ice cream waggons A and B that are going to supply ice cream on a beach that is 1km long. Suppose that there are 1,000 buyers who are located uniformly along the beach: there is one buyer per meter. Each buyer wants to buy one and only one ice cream per day and obviously will buy it from the waggon that is the closest; if the waggons are equally close a buyer will 'flip a coin' when deciding from which waggon to purchase the ice cream. The objective of each seller is to maximize sales.

a) Suppose that waggon B is located right in the beginning of the beach, is it a good strategy for the seller A to locate at the end of the beach? What is A's best response to the location of waggon B? *(Here a strategy is a choice of location, if B is located at 0, what is A's most profitable location?)*

b) What is a Nash equilibrium location of the waggons on the beach?

Problem 3. ADVERTISEMENT GAME.

Two cereal companies A and B advertise their products on TV. Each TV commercial increases general interest in healthy life style and raises sales of both companies. Profits as a function of each company's spending on advertisement are given by:

$\pi_A = (85 - x_B)x_A - 2x_A^2$

$\pi_B = (100 - x_a)x_b - x_B^2$

where x_A and x_B is spending by each company. Find how much each company will spend on advertisement in Nash equilibrium of a simultaneous game. *(Hints: at first you need to find best response functions. For A max π_A with respect to x_A, taking x_B as given, find x_A as a function of x_B - this is BR function of company A to advertisement expenditure of company B. Do the same for company B. Find x_A and x_B that satisfy both BR'es.)*

[1] $MR = 100 - 2q = 20 = MC$ gives monopoly output $q = 40$.