An Alternative Approach to the Analysis of Taxation

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Because commodities as transacted are complex, tax statutes could not cover all margins subject to optimization. A tax will induce, then, substitution within the commodity away from the taxed attributes and into the others. The results of a test on cigarettes are consistent with our prediction that the effects of unit and ad valorem taxes will differ both from each other and from those predicted by the conventional model. It is shown that, although the market will adjust in numerous changeable characteristics, the adjustment is constrained by the condition that the sum of the dollar value of the inefficiencies and of the tax paid is minimized.

The Dog and the Accountant

A dog whose fleas had become unbearable sought the advice of an accountant.

"Can I claim these fleas as exemptions?" he asked.

The accountant smiled and shook his head. "Under certain conditions," he explained, "ticks can be claimed as blood relatives. Fleas, on the other hand, are considered luxuries and, far from being deductible, are actually subject to tax."

The following day, the dog had no more fleas; or he didn't declare them, which comes to the same thing.

Moral: There can be no taxation without misrepresentation.

[J. B. HANDELSMAN][1]

This paper offers an extension of the traditional approach to the analysis of taxation capable of explaining observations attributable to taxation which were not previously explicable. Underlying the change in approach are the premises that *every* commodity is composed of a set of characteristics whose levels are economically determined and that no tax will be so comprehensively specified as to control all the characteristics of a commodity. Market adjustments to a tax will take place not only in the quantity of the good but also in the make-up of its characteristics.

By standard analysis, a tax on the tonnage of ships would simply lead to a substitution resulting in lower tonnage. Such analysis, however, is incapable of explaining why "shipowners had vessels built excessively long, deep and narrow, 'all more or less crank' [i.e., poor in seaworthiness]" (Lane 1964). In the above instance, the tax definition of tonnage (motivated, it seems by considerations of the costs of measurement) was basically proportionate to the length of ships and negatively related to the square of their width. Although deeper and narrower ships took a toll in handling, they did achieve from 1.1 to 1.4 tons of carrying capacity for each "ton" as measured for the purpose of tax.

In this paper it will be shown that, subject to the relevant constraints, the losses to market participants resulting from either unit or ad valorem taxes are treated as a magnitude to be minimized, rendering the adjustments in characteristics predictable.

I. The Setting of the Problem

A consumer's evaluation of a commodity rests on its various characteristics. The commodity "gasoline," for example, is demanded for its potential for generating energy, for its octane rating, and for its additives, as well as for such peripheral services as windshield cleaning. The choice open to consumers of commodities composed of many characteristics has been analyzed by Griliches (1961) in his empirical work on automobile prices and by Lancaster (1966) in his theoretical work on demand. We wish to proceed further by attempting to explain how the market will determine the levels of a commodity's characteristics in response to a tax.

For any given commodity tax, it is crucial to know how the statute defines the unit of the commodity, how that unit is measured, and how the tax is to be enforced. The unit for tax purposes may correspond to the market norm—gallons of gasoline, or packages of 20 cigarettes. Granted that the legal definition of the unit will certainly be more detailed, it is still highly unlikely to cover all revelant characteristics.[2] Furthermore,

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[2] The legal definition will almost inevitably become more detailed as inadequacies become evident. For example, the present tax-purpose definition of a cigarette will presumably expand eventually to cover the newly emergent "little cigars." Earlier, a maximum length for cigarettes had to be specified to counter the marketing of extra-long cigarettes, taxed at a constant rate per unit, which could subsequently be cut into standard lengths.

entire characteristics are subject to inclusion or exclusion with varying economic conditions. We seek to discover the nature and extent of these adjustments. As will become evident, the results depend critically on whether the tax is fixed per unit of the commodity or is ad valorem.

With a few explicit exceptions, we assume that prior to the imposition of a tax the market is competitive and free of distortions. A partial equilibrium framework is used throughout the analysis, and we abstract from possible feedbacks due to the effect of the tax on the level of income or on income distribution.

Because existing studies of the effects of commodity taxes employ demand and supply elasticities estimated prior to the tax, their results reflect only the change in volume of the commodity as a whole with no allowance for change in the mix of characteristics or any resultant effect on allocation. In a more careful analysis, this bias would be avoided by the treatment of different varieties as separate commodities, taking into account not only the individual elasticity of each but also their cross-elasticities, as noted by Harberger (1964, pp. 25–70). While this is a step in the right direction, it still does not allow for possible tax-induced changes in the characteristics of each variety. ("Cigarettes," for instance, may shift through a wide spectrum: the quality of tobacco and of its wrapping; the per-package retail services afforded for a given brand; the menthol or other components; and, of course, the length of the cigarettes themselves.) This paper explicitly takes into account this capacity for internal change within each commodity.

II. The Constant per Unit Tax

The traditional theory of taxation under competition is based on the comparison between the systems $D(P) - S(P) = 0$, and $D(P^T) - S(P^T - T) = 0$, where $D(P)$ is the market demand for a commodity, $S(P)$ is its supply, P is the price per unit before the tax, P^T after the tax, and T is the tax per unit. Misallocation occurs because resources shift away from the taxed industry into less highly valued alternatives. For the marginal unit the value of the misallocation is equal to the tax rate. Under common but rather restrictive assumptions, the economic waste as a result of the tax is measured by $\frac{1}{2}[\eta\varepsilon/(\eta + \varepsilon)](T/P)^2$ where η and ε are the respective demand and supply elasticities.

For a commodity composed of several characteristics, equality between the marginal valuation of the commodity as a whole and its marginal cost is a necessary, but not a sufficient, condition for competitive equilibrium. The marginal valuation (MV) of each of the characteristics must equal its marginal cost (MC).[3]

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[3] In what follows, we assume that each of the marginal valuations is falling and each of the marginal costs is rising.

Consider a commodity possessing n characteristics. At the pretax equilibrium $MC_i = MV_i$, $i = 1, \ldots, n$. Defining

$$MC = \sum_{i=1}^{n} MC_i \quad \text{and} \quad MV = \sum_{i=1}^{n} MV_i$$

for the commodity as a whole,[4] the equality for each characteristic implies also that $MC = MV$.[5] After a tax-induced reduction in number of units, but prior to any change in the levels of the characteristics per unit, the following equality will hold:

$$\sum MC_i^T + T = P^T = \sum MV_i^T. \tag{1}$$

The reduction in quantity of the commodity as a whole constitutes a downward move along each of the MC_i schedules and an upward move along each of the MV_i schedules, so that

$$MC_i^T < MC_i = MV_i < MV_i^T \quad \text{for all } i.[6] \tag{2}$$

Suppose now that the tax statute defines the commodity in terms of the pretax equilibrium levels of characteristics, $i = 1, \ldots, k$; but that characteristics $k + 1, \ldots, n$ are not specified in the tax statute. We can decompose (1) into

$$\sum_{i=1}^{k} MC_i^T + \sum_{i=k+1}^{n} MC_i^T + T = P^T = \sum_{i=1}^{k} MV_i^T + \sum_{i=k+1}^{n} MV_i^T. \tag{1'}$$

Given (1') and the inequalities in (2), prior to any change in the characteristics per unit of the commodity we have

$$\sum_{i=1}^{k} MC_i^T + T > \sum_{i=1}^{k} MV_i^T, \tag{3a}$$

and

$$\sum_{i=k+1}^{n} MC_i^T < \sum_{i=k+1}^{n} MV_i^T. \tag{3b}$$

Here (3a) relates to characteristics specified by the tax statute and (3b) to those which escape the constraint. The tax, strictly speaking, is imposed only on characteristics $1, \ldots, k$. Since the tax-inclusive cost of the first k characteristics exceeds their valuation, while the converse holds for the

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[4] To fix units of measurement we set the pretax level of each of the characteristics at one unit per unit of the commodity. Thanks are due John Floyd for correction of a previous error.

[5] An MV_i should be construed as the change in the total value of the commodity induced by a unit change in the ith characteristic and similarly for MC_i.

[6] We abstract from cross-effects among the characteristics in either cost or valuation functions which could conceivably reverse the inequalities.

other $n - k$ group, there is incentive to contract the former relative to the latter. Since the tax statute determines the level of the former per unit of the good, adjustment will have to take place by reducing the number of units produced while increasing the levels of the uncontrolled characteristics per unit of the good. Expansion of the untaxed characteristics constitutes an effective substitution against the tax. Adjustment will continue to the point where the inequality in (3a) and all of the $n - k$ components in the inequality in (3b) turn into equalities.

Since characteristics $1, \ldots, k$ are constrained to a maximum level by the tax statute and since there is basically no incentive to raise them, their level will stay unchanged and we can consider them as one. Characteristics $k + 1, \ldots, n$ are not constrained; but to simplify we shall assume that their relative levels are maintained. Let us call the taxed set of characteristics A, the untaxed B, and the commodity as a whole C.

The tax will cause a decline in the number of units of A demanded. The demand for B may increase or fall, depending on whether A and B are substitutes or complements, but that of B relative to A will have to increase.[7] While each unit of C will contain the same A as before, the tax will cause it to include more of B. In other words, the tax will induce an upgrading in the "quality" of C inasmuch as each unit will contain more B.[8]

The opportunity to modify the composition of the commodity reduces the tax burden implied in the fixed-quality model, and the equilibrium quantity will increase correspondingly. The "welfare triangle," therefore, is smaller than in the fixed-quality case. The change in resource allocation induced by the tax, however, is no longer confined to a reduction in quantity of the taxed good but now includes the costs of producing an inefficient mix of characteristics within the good.[9]

Thus participants in the market for the commodity suffer three kinds of losses from the tax: the tax payments,[10] the welfare triangle from re-

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[7] In a two-commodity world and holding individual incomes constant the demand for B per unit of A will remain unchanged only if they are perfect complements. Even though basically the tax will not affect aggregate income, it will affect its distribution. The individual income effects will not necessarily cancel out and may produce a perverse outcome. In addition, as noted by Gould and Segall (1969), interaction with other commodities could conceivably upset the results. Finally, the change in the quantities of A and B implies movements along the (pretax) supply schedules. The two supply elasticities may differ sufficiently to change the expense of A relative to B. None of these possibilities seems empirically important. In what follows, we abstract from all of them.

[8] The term "quality" is not used in a normative sense. Indeed, if B rather than A were being used as a unit of measure the "quality" would be reduced by the lesser proportion of A. It is used here as a shorthand notation for the quantity of given characteristics per unit of the commodity.

[9] A more complete discussion of these points in relation to a special case is presented in Section V, following.

[10] As already mentioned, we abstract from whatever benefits may arise from the use to which the tax is put.

duced production, and the loss of efficiency in production. Consumers and producers of the taxed commodity will benefit most by minimizing the sum of these losses. Our central proposition is that market forces will actually produce that result.

To demonstrate the proposition: consider the market for our commodity in a (taxless) economy in full competitive equilibrium. From all possible positions open to the economy, equilibrium is at the point at which the sum of consumers' and producers' surplus is maximized. Although most economic models consider one-dimensional goods, the fundamental properties of equilibrium are not affected when commodities have more than one characteristic.

With any change in the price of a factor used in producing the commodity, the economy will move to a new equilibrium. Suppose now a change occurs which has no effect on B but which raises by T the real cost of producing A. From the previous analysis of the effect of the tax (and abstracting from resultant differential changes in the rest of the economy) it is clear that the same equilibrium will be reached at the new factor price as would have occurred under the tax. Given the change in cost, the new equilibrium is again a position of maximum surplus, representing the smallest possible loss as compared to the prior situation. The market will react to the higher factor cost by economizing in its use (1) by contracting the output of the commodity itself and (2) by substituting toward those of its characteristics which use relatively less of the high-priced factor. In short, the market reaction is identical to that in the tax case; in both instances the adjustments constitute a minimization of the sum of the two cost components. Thus the proposition is demonstrated that following the imposition of a tax the market will move to minimize the sum of the tax paid and the welfare loss.

This proposition has been set out in terms of only two characteristics, but its logic is equally applicable irrespective of their number. At competitive equilibrium for any such commodity, economic surplus is maximized. Suppose an increase in the price of a factor used to produce some, but not all, of the characteristics takes place. The levels of each will be so adjusted that all the marginal conditions will be satisfied. The new equilibrium, subject to the higher factor price, will again be a position of maximum surplus, and the move to the second equilibrium will have entailed the smallest possible loss in surplus. Given the complete analogy of impact between a tax and an increase in factor price, the fundamental result enunciated above is entirely general.[11]

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[11] Such terms as welfare triangle, misallocation, inefficiency, distortions, and the like carry normative connotations. We have just shown, however, that they possess positive properties as the subject of market minimization. This terminology, ingrained in the literature, is retained here but is used only in its positive sense.

III. The ad Valorem Tax

Suppose an ad valorem tax of rate t is imposed on a commodity having n characteristics. The tax statute here defines the commodity in terms of the *minimum* levels of characteristics $1, \ldots, k$. The after-tax equilibrium prior to any change in characteristics can be written as:

$$(1 + t) \sum \mathrm{MC}_i^t = P^t = \sum \mathrm{MV}_i^t, \tag{4}$$

which is the counterpart of (1) for the constant-unit tax. Expression (4) can be decomposed, as is (1'), into the constrained and unconstrained components:

$$(1 + t) \sum_{i=1}^{k} \mathrm{MC}_i^t + (1 + t) \sum_{i=k+1}^{n} \mathrm{MC}_i^t = P^t = \sum_{i=1}^{k} \mathrm{MV}_i^t + \sum_{i=k+1}^{n} \mathrm{MV}_i^t \tag{4'}$$

Economic choice again determines both what characteristics will constitute a given commodity and their levels. A characteristic will be included in a commodity only if it is cheaper when part of the package than when sold separately. Since characteristics $k + 1, \ldots, n$ are not constrained, reduction of their levels will serve to evade some of the tax. Most important, these characteristics are taxed only so long as they are sold as part of the commodity.[12] Thus, if the pretax advantage of selling the $k + 1$st characteristic as part of the package exceeded that of selling it separately by less than $t x \mathrm{MC}_{k+1}$, it may be removed altogether from the package when the tax is imposed.

To simplify, let us again lump the first k characteristics into A and the remainder into B, and assume that A and B are consumed in fixed proportions. Suppose that, prior to the tax, a package was preferred. This is shown in figure 1, where D is the demand for the package, $S(P)$ is the supply of the package, $S(A)$ is the supply of one component, and $S(A + B)$ is the sum of the supplies of the two components (plus assembly, if necessary).

What would be the effect of an ad valorem tax? If the definition of the commodity for tax purposes is $A + B$, then the conventional analysis

[12] In at least some situations, substantial savings accrue to buying a package rather than its individual components. A study published in *Money Magazine* (1972) shows that while the retail price of a Ford Maverick was then $2,506.55 the retail price for its more than 10,000 parts, unpainted and unassembled, would have totaled $6,435.22. To this could be added the cost to the buyer of time spent in searching and buying.
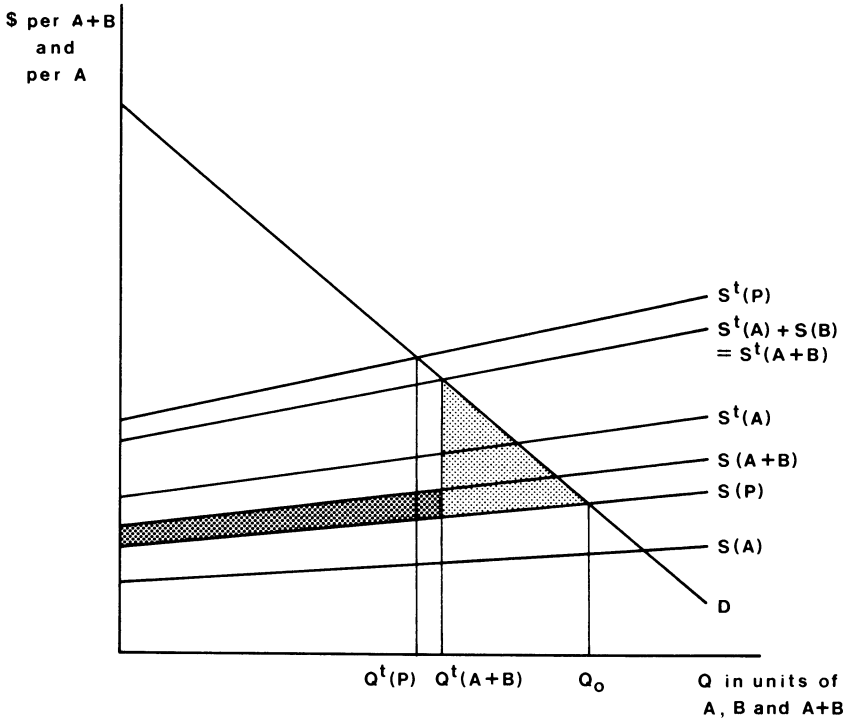
FIG. 1

holds. But if the definition covers A only,[13] the cost of the package rises relative to that of its components, since B is taxed only when sold as part of the package. $S^t(A + B)$ is the sum of the supplies of the components of which only A is taxed and $S^t(P)$ is the supply of the taxed package. As drawn, the former intersects the demand curve at a lower point and so is preferred. The price for $A + B$ paid by the consumer is less if he buys the components separately, so the after-tax quantity falls only to $Q^t(A + B)$ rather than to $Q^t(P)$, which would be the equilibrium quantity were the whole package taxed. As a result, the welfare triangle is also smaller than when quality is unchanged. But the reduced output is now produced at a resource cost according to $S(A + B)$ rather than to the lower $S(P)$, and this inefficiency adds to the welfare cost of the tax. The two components of the total welfare loss form the shaded areas in figure 1.

Tax revenue is now obtained from a larger number of units—$Q^t(A + B)$ rather than $Q^t(P)$; but as the tax base is smaller, the tax per unit is less

[13] Such would be the case where a new air-conditioned car is taxed at the same rate as one lacking that option and where separately purchased air-conditioning units are untaxed. For the tax to apply to all components of a car, the statute would have to cover each of many thousand parts, specifying for each a minimum standard of quality.

and we cannot determine a priori the direction in which the total tax payment will change. The after-tax price of the commodity as defined for tax purposes not only will be less than predicted by the constant-quality model but may actually fall below the pretax price.[14]

The analysis does not change in any essential way when we drop the assumptions of only two attributes consumed in fixed proportion. An ad valorem tax will induce a reduction in all the unconstrained attributes; some will be retained as part of the commodity but at lower levels, while others will be dropped altogether.[15]

IV. The Distinction between Unit and ad Valorem Taxes

Provided both that the appropriate tax rates are imposed and that quality is held constant, when a commodity is taxed the subsequent resource allocation and tax revenue will be the same regardless whether the tax is constant per unit or ad valorem. But when quality is not fully controlled, the effects of the two types of taxes differ radically. The excise tax induces a substitution of quality for quantity, resulting in a greater increase in the price per unit[16] than predicted by the constant-quality model. The ad valorem tax induces a reduction in quality and a lower after-tax price than predicted. Thus the statement by Due (1959, p. 298, italics added) that "this difference [between ad valorem and specific taxes] does not alter the *basic* initial reactions or the *ultimate* distribution of burden" has to be rejected if our hypothesis is valid.

It is evident that when quality is adjusted, neither the per unit nor the ad valorem tax remains truly constant. The former varies because the taxable unit has been changed. The latter is transformed into a variable tax because the value of the product is altered. In each case, the motivation for the quality change is a lower tax per unit of service.

The relation between the two taxes can be traced to an essential feature of commodities transacted in the market. Consider again a commodity possessing n characteristics and assume that the tax statute defines the product in terms of the first k. If deletion of characteristics were costless, the reaction to either a per-unit or an ad valorem tax would be the same. Characteristics $k + 1, \ldots, n$ are untaxed by the excise tax; their removal

[14] The corporate income tax is fundamentally an ad valorem tax. When such a tax is imposed or raised, it can be evaded in part by switching to corporate financing based more on debt and less on equity. The risk borne by equity holders will then increase. The measured rate of return to equity, reflecting this risk, might exceed the change in the tax rate, giving the impression of more than 100 percent shifting.

[15] It may appear that a general sales tax on final consumer commodities (or a uniform income tax) will avoid these problems. However, in addition to the difficulty of taxing home production, there is an incentive for employers to provide their employees with consumption goods such as air conditioning and uniforms—classified under the tax as intermediate and not subject to tax—thereby still partly evading the tax.

[16] The unit referred to is that defined for tax purposes.

from the commodity will exempt them from the ad valorem tax; thus substitution toward these characteristics is implied in either case. We can predict a reduction in the consumption of the first k characteristics, and a relative increase in the consumption of the others.

Suppose now that the ad valorem tax is set at such a rate that the amount realized after the elimination of characteristics will equal that attainable under a unit tax. The two taxes would now have *identical* effects on resource allocation. If we measure the commodity in units as defined by the tax statute, the tax-inclusive price will be lower for the ad valorem tax, but this is simply because the characteristics $k + 1, \ldots, n$ are sold separately.

Of course, it is invalid to assume that the deletion of characteristics will be costless, since they would not originally have been incorporated in the commodity unless this offered advantages. An ad valorem tax will still lead to a reduction of the levels of all the $k + 1, \ldots, n$ characteristics per unit of the good. While some characteristics may be completely eliminated, others will merely be reduced. The two taxes, then, will result in quite dissimilar effects on resource allocation as well as in different observed prices and tax revenues.[17]

V. Tax and Durability

In this and the following section, the analysis will be confined to a commodity with one continuously variable attribute—durability. This simplification not only will clarify some of the foregoing conclusions, but will help in the analysis of additional relevant issues. Thus, for the commodity selected for illustration—light bulbs—we consider only the variable of durability. The bulbs are assumed to be homogeneous in such dimensions as strength and color, and while they last they provide a constant-quality flow of services (the one-horse-shay model).

Consumers, we assume, value the flow of light services but not durability per se. A consumer, then, would be willing to pay the same price for 12 1-month bulbs as for a single 1-year bulb. This valuation scheme becomes plausible if we conceive of the consumer as renting the bulbs, that is, purchasing the light service rather than the product. The cost of changing bulbs is then transferred to the seller and can be incorporated in his cost function.[18] Under these conditions the durability offered in the market will be that for which the price of light services is lowest, satisfying the condition that the cost of an infinitesimal increase in volume of service will be the same whether obtained by an infinitesimal increase in the number of bulbs or in their durability.

[17] It would be interesting to discover under what conditions one type of tax would be selected over the other.
[18] We also assume that for any given volume of service there is always a single durability for which the cost is lower than for any combination of durabilities.

Now consider an excise tax on light bulbs. We assume that the tax statute does not constrain durability. This is not unreasonable, since in our model durability is not an explicit feature of bulbs as only one durability is provided by the market at a point in time. A tax which is constant per bulb raises the marginal cost of bulbs by the amount of the tax but does not affect the marginal cost of durability. The least-cost method of obtaining any given output of light services after the tax is to increase durability to the point where equality on the two margins is restored.

When we compare the constant durability model—fundamentally the accepted model—with the unconstrained durability model, several major differences emerge. With the removal of the constraint, the tax-inclusive cost of service will be lowered and the quantity of services increased. In general, competition drives the price to the lowest level consistent with cost. Here a move to a higher durability reduces the tax-inclusive cost per unit of service and competition will guarantee that more durable bulbs will actually be provided.

Received theory predicts that an excise tax will result in an increase in price not larger than the rate of tax. However, since the increase in durability will itself tend to increase the price of light bulbs, the actual increase in price per light bulb may easily exceed the rate of tax.[19]

Much of the content of the above results can be presented diagrammatically. In figure 2, the units on the horizontal axis represent light services and on the vertical axis, dollars per unit of service. Since the demand for service is independent of durability, a single demand curve suffices. For each durability, however, there is a different cost curve. To simplify graphic exposition, it is now assumed that cost curves for varying durabilities do not intersect. We denote by $S(x)$ the cost curve for durability x. Two such cost curves are drawn in figure 2. Suppose that the lower one, $S(2)$, provides light services at lower resource cost than any other durability, either shorter or longer. Before taxation, then, $Q(2)$ and $P(2)$ are the market quantity and price.

A tax of T per light bulb is now imposed. Each of the $S(x)$ curves will shift upward by amounts inversely proportional to x; the tax per unit of service on a 4-month bulb is half that on a 2-month bulb. The lowest curve will now be one of longer durability; the higher the tax, the greater

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[19] Following Harberger (1964), one could alternatively argue that the tax is actually imposed not on a single commodity but rather on a whole array of commodities where each durability constitutes a separate commodity. It is clear that the tax-induced increase in the consumer price for each would not exceed the rate of the tax. But the elasticity of substitution in consumption between alternative durabilities is such that the old durability is entirely replaced. This is formally correct but does not seem useful. In most empirical studies of taxation, the level of aggregation of the taxed commodity is quite high, indicating lack of awareness of the problem of tax-induced substitution within what is classified as a homogeneous commodity.
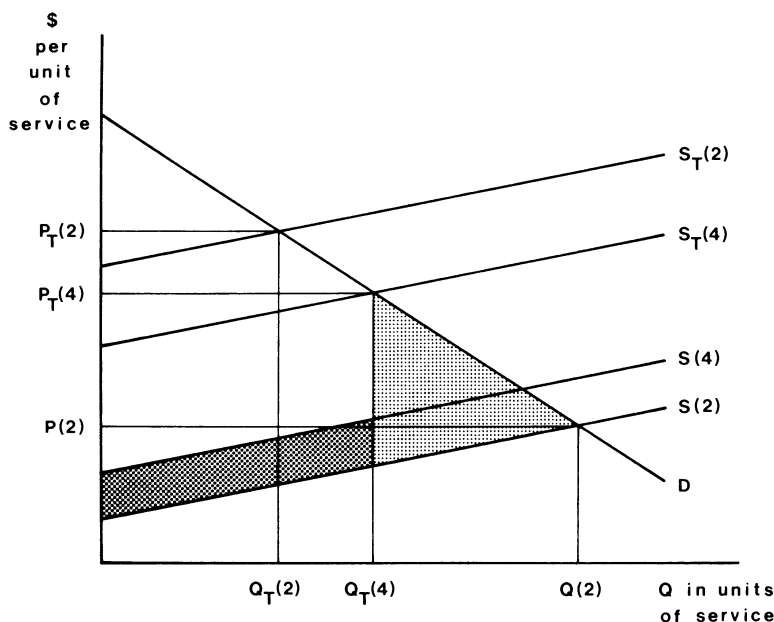
FIG. 2

that durability. We denote by $S_T(x)$ the tax-inclusive cost of light services obtained from bulbs of durability x, and suppose that among all these $S_T(4)$ is lowest.

After the tax, $Q_T(2)$ and $P_T(2)$ are the equilibrium quantity and price if durability is constrained to 2. But if durability is not constrained, the final price per unit of service will be lower and the final quantity of services higher, since $S_T(4)$ lies below $S_T(2)$. Note, however, that the price observed in the market is not per unit of service but per bulb. Thus even though $P_T(4) < P_T(2)$ and $P_T(2) - P(2) <$ tax, the increase in the price per bulb may exceed the tax rate because the after-tax bulb is more durable. Indeed, if the supply curves are perfectly elastic, the increase in the price per bulb will *necessarily* exceed the rate of tax.

Let us now turn to tax receipts. In figure 2 these are represented by the product of the after-tax market quantity and the difference between the after-tax market price and the pretax cost. It is easy to see that tax receipts may go up or down when we compare the constrained with the unconstrained durabilities. For instance, if the tax is originally set to yield maximum tax revenue while constraining durability to its pretax level, when the constraint is relaxed the substitution towards higher durability will necessarily result in a revenue loss.[20] On the other hand, if the im-

[20] Finding the tax rate that yields maximum revenue is equivalent to finding the largest rectangle that can be inscribed between the demand and supply curves to the left of their intersection. Since higher durabilities reduce the area between the two curves, the new largest rectangle has to be smaller than the old one.

posed tax rate is just prohibitive, it will not remain so for certain longer durabilities, and tax revenues will then increase from zero to some positive value.

What are the pertinent welfare costs for the two alternatives? With no change in durability, the deadweight loss due to the tax is the conventional triangular area between the demand curve and the supply curve $S(2)$ from $Q_T(2)$ to $Q(2)$. When durability is permitted to adjust, the price paid by consumers is lower and the quantity purchased larger than when durability is constrained. Thus the conventional welfare triangle is reduced to the shaded area between D and $S(2)$ from $Q_T(4)$ to $Q(2)$.

However, when durability is variable, misallocation is no longer restricted to that resulting from contraction of output. Two-month durability is most efficient since it provides light services at lowest resource cost, but in the process of partially evading the tax durability has now been increased to 4 months; thus the *entire* quantity is being produced inefficiently. The quantity $Q_T(4)$ could have been produced at the resource cost indicated by $S(2)$ rather than by $S(4)$. The shaded area between $S(4)$ and $S(2)$ up to quantity $Q_T(4)$ represents the value of the tax-induced excess resources used in producing light services.

What are the properties of the new equilibrium position? Together, producers and consumers search for that durability for which the effect of the tax is minimized: that is, for the real price that results in the smallest contraction of service output. In other words, market participants search for the durability that leads to the lowest after-tax price per unit of service. The tax paid per light bulb is still T, but now a bulb provides more services, thus the real tax is lower by a corresponding amount.

To depict further the properties of the after-tax equilibrium, figure 3 adds to the demand and supply curves of figure 2 the supply curves of 3-month bulbs, both before and after the tax and a new curve denoted H.[21] Each point on H represents a net-of-tax market equilibrium position if durability is constrained to a particular value. Thus, point 2 is the after-tax equilibrium to suppliers if durability is constrained to 2 months. Similarly for points 3 and 4. Points 2′, 3′, and 4′ are the corresponding points of demand. That is, given the tax and the constrained durabilities, these are the optimal points to consumers. The distance between 2 and 2′, for instance, is the tax per unit of service for a bulb of 2-month durability. The H curve is tangent to $S(2)$ at point 2, since $S(2)$ is the lowest supply curve.

As we move from point 2 through higher durabilities, output is produced at increasing resource cost. In the durability range from 2 to 4 months, however, this increase in cost is more than matched by tax savings. In that range, then, the price paid by consumers is declining, the quantity purchased rises, and the welfare-loss triangle shrinks correspondingly.

[21] The discussion of figure 3, and a following related consideration of subsidies, were stimulated by a conversation with Al Harberger.
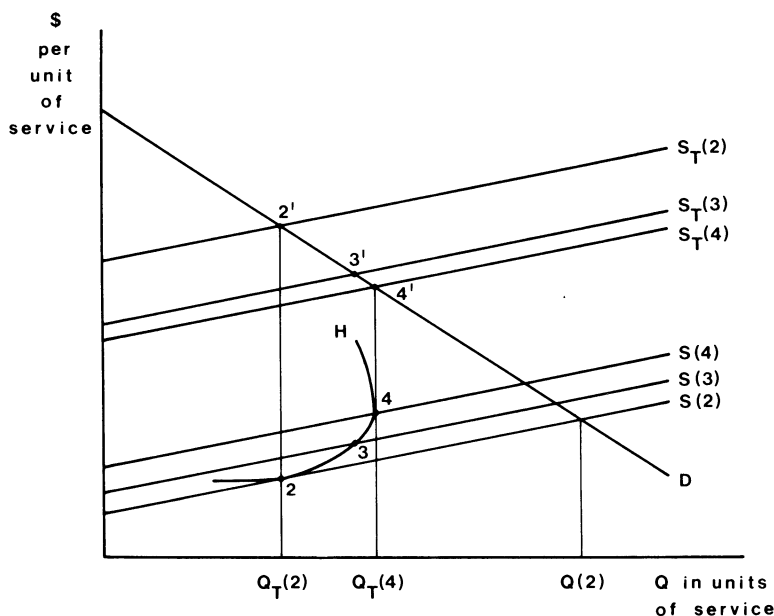
FIG. 3

Beyond point 4 the tax saving of increased durability is less than the extra resource cost. At point 4, where durability is identical to that chosen in the absence of constraint, the H curve is vertical and the incremental gain from a further saving in taxes is exactly matched by an increase in cost of production. That point, then, satisfies the first order condition for a minimum. Since point 4 is market determined, the earlier proposition that the market will minimize the dollar value of the sum of welfare losses and tax paid is again demonstrated, though from a different angle.[22]

VI. Subsidies

It may appear that a discussion of subsidies at this point is superfluous: Aren't subsidies the "mirror image" of taxes? Why would not a subsidy simply reverse the signs of the tax case? It is true that a tax is a negative subsidy; the effect of the former is to reduce market quantity and of the latter to increase it. But the effect is not symmetric with respect to resource allocation. *Both* lead to positive welfare triangles and to inefficient moves from the optimal mix of characteristics into mixes which use up more

[22] In the case just discussed, would an ad valorem tax affect durability, or would it be "neutral"? If the tax is on bulbs, but not on their installation, the buyer has a stronger incentive to install his own bulbs. As the cost of installation relative to bulbs is then lower, durability will be shortened.

FIG. 4
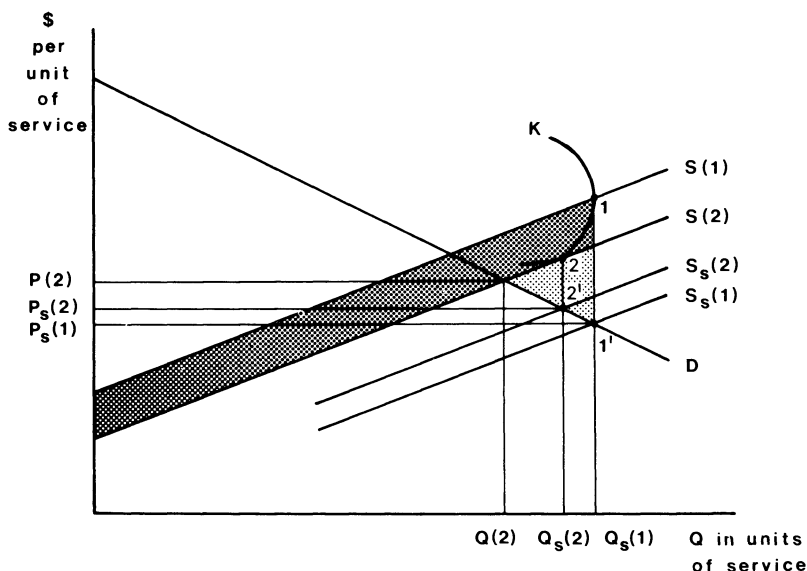
resources. In the remainder of this section we examine this aspect of the effects of a constant per unit subsidy on a durable commodity.

In addition to directly increasing quantity, a subsidy will lead to an added expansion in quantity because of substitution against durability, since the price per unit of service is thus reduced. In figure 4, $S(2)$ and D are again the same as in figure 2. A subsidy, S, is granted per light bulb of whatever durability. If durability is constrained to remain at 2 months, the price paid by consumers per unit of service will decline to $P_S(2)$ and the quantity will increase to $Q_S(2)$. But for durabilities shorter than 2 months the subsidy per unit of service is higher, thus light service will be cheaper. Suppose that among bulbs of shorter life those of 1-month durability are best; $S(1)$ and $S_S(1)$ as drawn in figure 4 show that when the durability constraint is removed the price per unit of service will fall to $P_S(1)$ while the quantity of service consumed will increase to $Q_S(1)$.

In regard to welfare loss, since the market quantity of services is lower when durability is held constant, the removal of that constraint leads to a *larger* welfare triangle—the shaded triangle in figure 4. In addition, bulbs of 1-month durability provide light service at a higher resource cost than 2-month bulbs, consequently the entire output is produced inefficiently. The total resource cost of this inefficiency is the area between $S(1)$ and $S(2)$ to quantity $Q_S(1)$, which is also shaded in the diagram.

It is evident that if the rate of the subsidy just discussed equals the rate of the tax considered earlier, and given linear demand and cost

curves, the adjustment to the subsidy is larger than that to the tax.[23] If the tax and subsidy rates are the same, the changes which will occur in price and quantity under conditions of constrained durability are of the same size (in absolute terms). When durability is allowed to change, a downward move is induced along the demand curve by either the tax or the subsidy. This mitigates the initial effect of the tax but enhances that of the subsidy— which thus has the greater total impact.

More generally, a series of constant tax doses will result in successively smaller adjustments in the price and quantity of service. The intuitive explanation for this is that since each tax increment leads to the production of more durable bulbs for the market, the new levy has a smaller proportionate effect.[24]

The curve K in figure 4 is obtained similarly to H in figure 3. This curve, too, is tangent to S_2 and is vertical at output level $Q_S(1)$. If durability is constrained to 2 months, producers will operate at point 2 after the subsidy is granted. As they are permitted to produce shorter durabilities they will move along the curve K as far as, but not beyond, point 1. At that point the benefit from obtaining a larger subsidy by switching to a still shorter durability is exactly nullified by the increase in cost of the shorter durability. Point 1 produces the largest welfare triangle; as durability was reduced from 2 months to 1, the triangle was increasing continuously; but it then begins to shrink because still shorter durabilities lead to a decline in market quantity. At point 1, which will be chosen by the market in the absence of constraint, the sum of welfare loss and the (negative) tax paid is at a minimum—a small move leaves the area of the triangle unchanged while the change in welfare loss due to inefficient production is exactly matched by the change in subsidy.

Consider now a change away from point 2. Here, an infinitesimal reduction in durability will result in an increased subsidy, but the efficiency in production will not be affected. This, then, will lead to a fall in price paid by consumers, to a corresponding increase in output, and hence to a larger welfare triangle. It is evident, then, as stated earlier, that any move to a shorter durability must result in an unambiguous increase in welfare cost.

When it is possible to tighten the legal definition of the commodity with the objective either of increasing tax revenue or of reducing the dollar value of the misallocation, more information is needed to determine

[23] Of course, a simultaneous tax and subsidy of the same size will identically cancel out. Here the starting point of the subsidy is the end point of the tax. In the textual comparison, the starting point for both is the same.

[24] Note, however, that the observations in empirical work typically correspond to those on light bulbs rather than on light services. It is not entirely clear, then, that the observed price increases will continue declining with successive equal tax doses; a result which anyhow depended on the assumption of linearity.

which way the tightening will work in the case of a tax. With a subsidy, on the other hand, a move to prevent substitution among qualities will always reduce both subsidy payments and misallocation.[25]

VII. A Test of the Hypothesis

Our fundamental hypothesis is that the effect of a tax on the price of a commodity, as measured, will differ from that indicated by conventional analysis. Specifically, we predict that a tax which is constant per unit will increase consumer price, as normally measured, more than is predicted by the received model, whereas an ad valorem tax will raise price less than a comparable per-unit tax.

Consider also the "mark-up" hypothesis which states (inconsistently with demand theory) that when the price to the seller is increased by whatever reason, he will raise the retail price by a larger amount. We argue that the effect produced by an ad valorem tax will differ radically from that of a constant per unit tax. Data on the effects of the two taxes may enable us to test the received model and also to choose between the mark-up hypothesis and ours.

Reported in the following paragraphs are tests of the hypotheses using data on cigarettes.[26] Starting in 1954, the Tobacco Tax Council (1972) has compiled and reported the retail price of cigarettes on a state-by-state and year-by-year basis.[27] The "prices" are actually within-state indices using a set of fixed weights for all states in all years. Weights are given to three types of cigarettes—regular, king, and filter (both 85 mm and 100 mm)—and to three types of sale—single pack over the counter, vending machine, and carton lot.[28]

Cigarettes provide an almost ideal setting for testing our hypotheses. The bulk of cigarettes originate from around North Carolina and are distributed nationally. Apart from the effect of shipping costs (which is possible to control for), the observed price differences at a point in time are presumably due largely to state taxes.

While a federal tax remained constant at 8 cents per package for the entire period, state taxes varied greatly within states and over time. In 1972, the most recent year used, these ranged from 2 cents per package in North Carolina to 21 cents in Connecticut. In the same states the taxes had been, respectively, 0 and 3 cents in 1954.

[25] Incidental to the central theme of this paper is the hypothesis that the quality constraints actually imposed would be more severe for a subsidy than for a tax.

[26] Attempts to test our hypothesis against data on gasoline and liquor are reported in the Appendix.

[27] For most years, the data are given as of November 1. For 1957, 1958, and 1959, the date used is October 1.

[28] Information provided in a letter from the Tobacco Tax Council.

TABLE 1

CIGARETTE REGRESSION RESULTS*

Variable	Coefficient	Standard Error	t-Value
Constant............................	14.801	0.201	73.564
Tax (in cents per package)	1.065	0.011	101.439
New Hampshire dummy	−0.072	0.014	−5.078
Distance (in 100 miles)	0.011	0.004	3.072
$R^{-2} = .93$			

* A set of year dummies included in the regression showed an upward drift from − 1.9 (cents per package) in 1954 to 0.8 in 1970 declining back to zero (the "base" year) in 1972.

It seems reasonable to expect that the supply elasticity of cigarettes to a single state is very high. Thus if the conventional model is correct, a 1-cent difference in tax per package between two states will result, *ceteris paribus*, in close to a 1-cent difference in price. Using our model, we predict a price increase larger than 1 cent if the tax is constant per unit and a smaller increase if it is ad valorem.

Table 1 presents results from a regression based on 931 observations.[29] The dependent variable is the retail price, deflated by the consumer price index for October of the given year. The coefficient of the tax (also deflated by the consumer price index) indicates that a 1-cent increase in tax raises retail price by 1.065 cents, which is significantly larger than one, as the t-ratio for that test exceeds 6.[30] The received notion that the price will increase at most by the amount of the tax can be rejected.[31]

In New Hampshire the tax is ad valorem. The regression coefficient for the dummy variable for that state, measuring the differential from other states with the comparable unit tax, is −0.072 and its standard error is 0.014. The retail price in New Hampshire, then, is significantly less than that predicted for a state with a comparable per package tax,[32] and the hypothesis can be rejected that comparable per unit and ad valorem taxes will have identical effects.[33] Also rejected is the mark-up

[29] These are one observation each for 19 years (1954–72) for the District of Columbia and for each of the 48 continental states. Alaska and Hawaii, for which data are available for only part of the period, were deleted from the sample because of the ambiguity of the distance measure. Note that Hawaii imposes an ad valorem cigarette tax at the wholesale level. Data sources are: for road distances between largest metropolitan areas, Rand McNally *Road Atlas* (1972); for cigarette prices and taxes, Tobacco Tax Council (1972); for consumer price index, U.S. Bureau of Labor Statistics (various years).

[30] We would expect within states a tax-induced shift toward higher-priced cigarettes such as 100 mm and those from vending machines. As a result, the average price per package should have risen. Since average weights are fixed in the data, the regression coefficient does not reflect this phenomenon.

[31] The cost of financing the tax in terms of extra interest can account for only a small fraction of the amount by which the coefficient differs from unity.

[32] The size of New Hampshire makes it unlikely that cigarettes would be tailor-made for it. The scope for removing characteristics is quite limited, then.

[33] In scatter diagrams for individual years the slope of the unit tax, fitted by free hand, is consistently higher than one, and New Hampshire has consistently the largest negative deviation.

hypothesis, as it fails to predict differential effects from the two types of tax. Finally, the coefficient of the distance variable, inserted to control for the differential shipping costs from North Carolina, is positive, as expected, and significant.

VIII. Final Comments

If economists are set to offer policy prescriptions, the preceding analysis warns of a serious pitfall. Our model, and our results, cast doubt on the notion held by many economists with respect to second-best solutions— that taxes can be used to correct distortions—as we argue that a tax will have additional "side effects."

For instance, a gasoline tax is often regarded as a means to combat air pollution. Since the tax is actually constant per gallon, market substitution will occur toward high-octane gasoline which has become relatively cheaper. But high octane ratings are produced by lead, now believed to be a major air pollutant, and the tax which results in a higher per-gallon use of lead may actually have increased the absolute level of pollution. The asserted purpose of the tax thus may have backfired.[34]

While taxes do affect resource allocation, the common normative statement that they misallocate resources makes little sense. If that is all they do, why would they ever be imposed? Until we can spell out the objectives of policymakers and the constraints they face we cannot attach meaning to terms such as misallocation or distortion. What we can do, however, is to determine the exact nature of any tax so that we can more accurately predict its effects.

Let us briefly summarize the thrust of the argument. Commodities as transacted in the market are complex, and the margins with respect to which optimization takes place are numerous. Because commodity tax statutes will not generally cover all these margins, any tax will induce multiple changes not only in resource allocation away from the taxed commodity and into others but also in the "quality" of the commodity and how it is transacted, a substitution away from the taxed attributes and into the others.

Adjustment in the face of incomplete specification of commodities implies upgrading of quality in the case of a unit tax and downgrading in that of an ad valorem tax. A test of the effect of taxes on cigarettes refutes the conventional hypothesis that quality will not change with a tax.[35] The results are rather consistent with our model, which claims that the effects of unit and of ad valorem taxes will differ both from each other and from those predicted by the conventional model.

[34] Plott (1966, pp. 84–87) arrives at a similar result for a similar reason.
[35] The results for gasoline and liquor are also consistent with our model.

The two taxes produce different effects because of the different specification of what, exactly, is being taxed. Given the definition (and enforcement) the market will adjust with respect to each of the changeable characteristics. Although these changes are bound to be numerous and will differ depending on the type of tax, their extent is sharply constrained. They will be carried only to the point that minimizes the sum of the dollar value of the resultant inefficiencies and of the tax paid.

Appendix

Tests for Gasoline and Alcohol

Gasoline.—An attempt to test our hypothesis using data on gasoline taxes produced mixed results, vaguely encouraging but quite erratic. The main problems seem to be that (1) at any moment the wholesale price differs widely across markets, and (2) identification is hard to establish.

The data used are reported by the monthly magazine, *National Petroleum News* (1971), where for several years monthly observations are given on the wholesale price, tax, and retail price of regular gasoline in 55 cities.[36]

Regressions were run for 4 months in 1971. The dependent variable was retail price per gallon and the independent variables were wholesale price and tax. With unlagged independent variables the coefficient for tax is 0.78 (0.078 SE) and for wholesale price 1.18 (0.031 SE). The forces that make us expect a "large" coefficient for the tax variable apply to other cost components.[37] The coefficient for the wholesale price variable, at least, is comforting.

The variability in the wholesale price may well be due to differing demands among cities, while our hypothesis is in terms of shifts of cost schedules rather than shifts along them. Thus one cannot accept the above results at face value. When the two independent variables were lagged in an attempt to assure that the direction of causality in the data is from the independent to the dependent variable, the coefficient for the tax became 1.05 (0.23 SE) and that for the wholesale price 0.29 (0.09 SE).

It is clear that a more complete model and possibly better data are required to obtain satisfactory results for gasoline.

Alcohol.—The data used are from a secondary source.[38] A price breakdown is provided of 12 distilled spirits in New York State for 1971. These data enable us to run a regression in which the retail price per case is the dependent variable and tax per case and wholesale price (net of tax) are the independent variables. Since measurement of one of the observations (imported brandy) presented problems, regressions were run with and without it. With the entire 12 observations the tax coefficient was 1.31 (0.40 SE) and that for the wholesale price 1.30 (0.15 SE). With 11 observations the respective coefficients were 1.51 (0.43 SE) and 1.26 (0.14 SE).

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[36] "Regular" gasoline does not sufficiently define the commodity; octane ratings and other characteristics differ among regions and brands. I was unable to correct for these, however.

[37] Therefore, the two coefficients should be approximately equal, but here the difference between them is significant.

[38] Daniel Wisecarver (1974) kindly permitted me to use his data from an unpublished paper (tables 1 and 6).

The tax coefficients are larger than one, but not significantly so, and they are of the same order of magnitude as the coefficients for the wholesale price variable. The results, while not overwhelming, tend to confirm the hypothesis.[39]

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[39] In a study by Niskanen (1963) one of the findings is that in the taxation of alcoholic beverages consumers bear more than 100 percent of the tax. This offers additional confirmation of the hypothesis.