Economics 460 MIT answer Key

1. Sparky has the utility function $u = xy^2$ and a budget constraint $M = P_x x + P_y y$. Initially $M = 36, P_x = 2, P_y = 1$.

   (a) Find optimal x, y, and u

$$\begin{aligned} x &= \frac{M}{3P_x} \qquad y = \frac{2M}{3P_y} \qquad u = \left(\frac{M}{3P_x}\right)\left(\frac{2M}{3P_y}\right)^2 = \frac{4M^3}{27P_xP_y^2} \\ M &= \sqrt[3]{\frac{27P_xP_y^2u}{4}} \\ x &= 6, y = 24, u = 3456 \end{aligned}$$

   (b) Suppose $P_x = 1$, what is the new U?

$$x = 12, y = 24, u = 6912$$

   (c) Find CV

$$CV = 36 - 28.57 = 7.4$$

   (d) Find EV

$$EV = 45.35 - 36 = 9.35$$

2. Suppose an industry has 12 firms, each with the following marginal (private) cost function

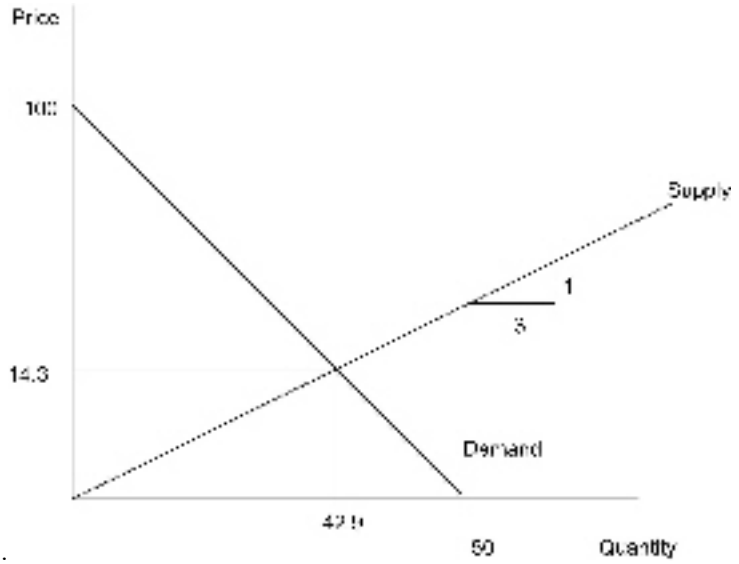
$$MC_i = 4Q_i \qquad (i = 1, ..., 12)$$

   and the market demand function is $Q^T = 50 - 0.5P$

   (a) Find the equation for the industry supply curve. ANSWER:

$$\begin{aligned} P &= MC = 4Q_i \\ Q_i &= \frac{1}{4}P \\ Q^S &= 12Q_i = 3P \\ &\quad or \\ P &= \frac{1}{3}Q^S \end{aligned}$$

   (b) Graph both supply and demand and find the equilibrium price and quantity



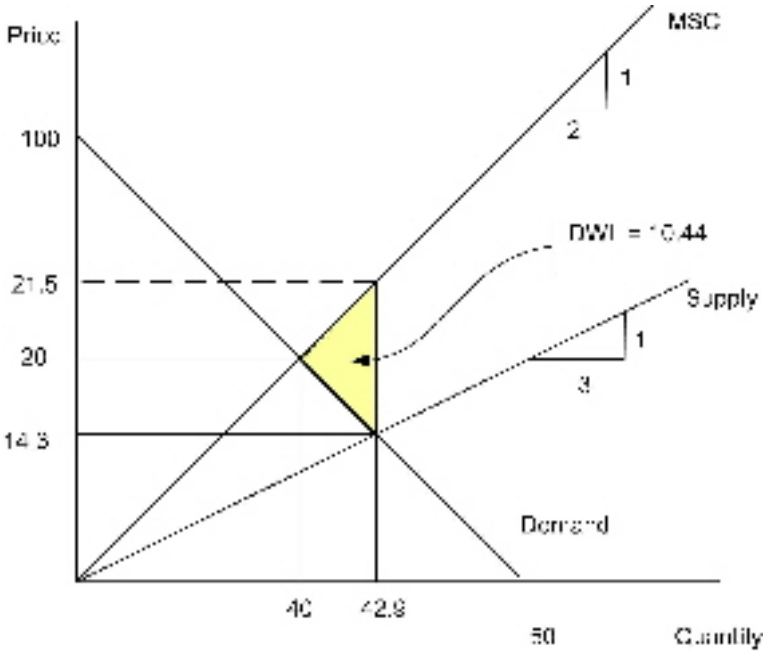
Graph:.

3. Using the information in problem 1, but now suppose that each firm's production causes external damage (pollution). The marginal external cost per firm is $MEC_i = 2Q$

   (a) What is the marginal social cost per firm $(MEC + MC)$? , what would be the supply curve?

   (b) Solve for the equilibrium price and quantities. Graph your results.

   (c) Using your results from problem 1, calculate the net welfare cost when firms DO NOT take the MEC into account


Price
100
21.5
20
14.3
MSC
1
2
DWL = 10.44
Supply
1
3
Demand
40
42.9
50
Quantity

ANS: b and c

4. Suppose we have three people who have different willingness to pay schedules, which are

$$\begin{array}{ll} A & MWTP = 100 - Q \\ B & MWTP = 110 - 1.1Q \\ C & MWTP = 120 - 1.2Q \end{array}$$

Further, the marginal cost of the good is $MC = 10 + .5Q$

(a) If this good is a"public" good, aggregate the MWTP schedules and calculate the socially optimal quantity. What is each person's MWTP for this quantity?

$$\begin{aligned} MWTP &= 330 - 3.3Q \\ MWTP &= MC \\ 330 - 3.3Q &= 10 + .5Q \\ 320 &= 3.8Q \\ Q &= 84.2 \\ A &= 15.8, B = 17.38, C = 18.96 \end{aligned}$$

(b) If this good is a private good, aggregate the MWTP in the appropriate manner and solve for the socially optimal quantity and price. How much of the good does each person consume?
ANSWER: first replace all the MWTP's with P and re-write to isolate the $Q's$ and then sum:

$$\begin{aligned} Q^T &= Q_A + Q_B + Q_C = 300 - 2.74P \\ & \quad or \\ P &= 109.5 - 0.365Q \end{aligned}$$

Then solve for equilibrium:

$$\begin{aligned} 109.5 - 0.365Q &= 10 + .5Q \\ 99.5 &= 0.865Q \\ Q &= 115 \\ P &= 67.5 \\ A &= 32.5, B = 38.6, C = 43.96 \end{aligned}$$

5. A small town gets tap water from a stream. Demand for bottled water depends on quality of tap water. If tap water is clean, demand for bottled water is

$$P = 12 - 0.1Q$$

If the tap water is dirty, the demand for tap water is

$$P = 22 - 0.1Q$$

Bottled water is sold at cost and the price is \$2. The water gets polluted by emissions from a factory. It can be cleaned up for a one-time cost of $\$20,000$.

(a) Calculate the annual willingness to pay for clean water.
CS when water is clean is $CS = 500$,
CS when water is dirty is $CS = 2000$
Willingness to pay is the change in CS due to the dirty water, or

$$WTP = 2000 - 500 = 1500$$

(b) If interest rate is 10%

$$\begin{aligned} NPV &= \frac{WTP}{i} - 20000 \\ &= \frac{1500}{0.1} - 20000 = -5000 \end{aligned}$$

NO

(c) If interest rate is 5%

$$\begin{aligned} NPV &= \frac{WTP}{i} - 20000 \\ &= \frac{1500}{0.05} - 20000 = 10000 \end{aligned}$$

YES