

*The following problem is based on Chapter 8 of the Textbook and my lecture from July 13<sup>th</sup>.*

Consider the following pollution offset problem:

There are two firms, each with the same total savings functions:

$$TS_1 = 200 e_1 - (e_1)^2 \text{ and } TS_2 = 200 e_2 - (e_2)^2$$

The marginal savings functions, respectively, are

$$MS_1 = 200 - 2e_1 \text{ and } MS_2 = 200 - 2e_2$$

At receptor Station One, the diffusion coefficients are  $a_{11} = 0.5$  and  $a_{12} = 1$ . The target ambient level is 100. Therefore, at Station One, the emission constraint is:

$$0.5e_1 + e_2 = 100$$

- a) Determine the cost minimizing levels of emissions at Station One (hint: use the Lagrange method).

Now suppose that there is a second receptor (Station Two) that has the following emissions constraint:

$$2e_1 + e_2 = 120$$

Where the diffusion coefficients are  $a_{21}=2$  and  $a_{22} = 1$ ; with a target ambience of 120.

- b) Use the same approach as in part (a) to determine the cost minimizing level of emissions that satisfy Station Two.
- c) Carefully graph both constraints on a graph with  $e_1$  on the horizontal axis and  $e_2$  on the vertical axis. Indicate the region that satisfies both constraints.
- d) Label the solutions to (a) and (b) on your graph. Can you determine if each violates the other constraint? Is there a solution where both constraints are satisfied? If so, which receptor has the binding constraint?
- e) Suppose the government had initially granted emission permits to each firm: firm one was given 13.33 permits and firm two was given 93.33 permits. (These are the initial allocations that satisfy both constraints). Given the information in your answers for (a) to (d), determine who will sell permits and who will buy permits. What will be the price ratio of the permits that are traded?