

Kolstad Chapter 14 - Emission Permits vs Ambient Permits

Question 6

> restart ;

>

> $ms1 := 10 - 2 \cdot e1$;

$$ms1 := 10 - 2 e1$$

(1)

> $ms2 := 10 - 2 \cdot e2$;

$$ms2 := 10 - 2 e2$$

(2)

> $a1 := 0.5$;

$$a1 := 0.5$$

(3)

> $a2 := 1$;

$$a2 := 1$$

(4)

>

First solve for emissions permits (EPS): Total quantity of permits = 6

> $q := 6 - e1 - e2$;

$$q := 6 - e1 - e2$$

(5)

Solve for equilibrium e1 and e2

> $\text{solve}\left(\left\{\frac{ms1}{a1} - \frac{ms2}{a2} = 0, q = 0\right\}, \{e1, e2\}\right)$;

$$\{e1 = 3.666666667, e2 = 2.333333333\}$$

(6)

Find total Pollution in equilibrium under EPS

> $z := a1 \cdot e1 + a2 \cdot e2$;

$$z := 0.5 e1 + e2$$

(7)

> $zEPS := \text{subs}(\{e1 = 3.67, e2 = 2.33\}, z)$;

$$zEPS := 4.165$$

(8)

Find Permit Price under EPS:

> $\text{priceEPS} := \text{subs}(e2 = 2.3333333, ms2)$;

$$\text{priceEPS} := 5.3333334$$

(9)

Firm 2 sells 1.67 permits to Firm 1

PART C

Now find equilibrium under APS: $L1 + L2 = 4$ and, in equilibrium $L1 = a1 \cdot e1$ and $L2 = a2 \cdot e2$

Solve $MS_1/a_1 = MS_2/a_2$ and $L = a_1 e_1 + a_2 e_2 = 4$

$$\begin{aligned} > L := 4 - a1 \cdot e1 - a2 \cdot e2; \\ & \qquad \qquad \qquad L := 4 - 0.5 e1 - e2 \end{aligned} \tag{10}$$

$$\begin{aligned} > \text{solve}\left(\left\{\frac{ms1}{a1} - \frac{ms2}{a2} = 0, L = 0\right\}, \{e1, e2\}\right); \\ & \qquad \qquad \qquad \{e1 = 3.600000000, e2 = 2.200000000\} \end{aligned} \tag{11}$$

Total Pollution under APS

$$\begin{aligned} > z_{APS} := \text{subs}(\{e1 = 3.6, e2 = 2.2\}, z); \\ & \qquad \qquad \qquad z_{APS} := 4.00 \end{aligned} \tag{12}$$

Permits demanded by Firm 1 and Firm 2

$$\begin{aligned} > L1 := a1 \cdot e1; \\ & \qquad \qquad \qquad L1 := 0.5 e1 \end{aligned} \tag{13}$$

$$\begin{aligned} > L2 := a2 \cdot e2; \\ & \qquad \qquad \qquad L2 := e2 \end{aligned} \tag{14}$$

Equilibrium permits/firm under APS

$$\begin{aligned} > L1_{\text{equilib}} := \text{subs}(e1 = 3.6, L1); \\ & \qquad \qquad \qquad L1_{\text{equilib}} := 1.80 \end{aligned} \tag{15}$$

$$\begin{aligned} > L2_{\text{equilib}} := \text{subs}(e2 = 2.2, L2); \\ & \qquad \qquad \qquad L2_{\text{equilib}} := 2.2 \end{aligned} \tag{16}$$

$$\begin{aligned} > \text{price}_{APS} := \text{subs}(e2 = 2.2, ms2); \\ & \qquad \qquad \qquad \text{price}_{APS} := 5.6 \end{aligned} \tag{17}$$

Firm 1 sells 0.2 permits to Firm 2

There are two firms, each emitting 20 units of emissions, which meets the standard set by the old CAC regulation. The government has decided to replace the CAC with a pollution offset program. There are two receptors in the region and, at the current emissions, both read 120 on the pollution index. 120 is the maximum allowed at either receptor under the new program.

When the two firms increase their emissions, their total savings is given by the following functions $TS_1 = \ln(e_1)$ and $TS_2 = 2\ln(e_2)$ {where \ln is the natural logarithm}

The marginal cost functions are, respectively, are $MS_1 = 1/e_1$ and $MS_2 = 2/e_2$

At receptor Station One, the diffusion coefficients are $a_{11} = 4$ and $a_{12} = 2$. Therefore, at Station One, the emission constraint is: $4e_1 + 2e_2 = 120$

a) Determine the cost minimizing levels of emissions at Station. [ANS: \$e_1 = 10\$ \$e_2 = 40\$](#)

At the second receptor, Station Two, the diffusion coefficients are $a_{21} = 3$ and $a_{22} = 3$; with a target ambience of 120.

- b) Use the same approach as in part (a) to determine the cost minimizing level of emissions that satisfy Station Two. [ANS: \$e_1 = 13.3\$ \$e_2 = 26.7\$](#)
- c) Carefully graph both constraints on a graph with e_1 on the horizontal axis and e_2 on the vertical axis. Indicate the region that satisfies both constraints.
- d) Label the solutions to (a) and (b) on your graph. Can you determine if each violates the other constraint? Is there a solution where both constraints are satisfied? If so, which receptor has the binding constraint? [ANS Receptor2 is the only binding constraint.](#)
- e) Suppose the government had initially granted 20 emission permits to each firm: Given the information in your answers for (a) to (d), determine who will sell permits and who will buy permits. What will be the price ratio of the permits that are traded? [ANS: Firm 1 will trade 6.7 permits to firm 2 at a ratio of 1:1](#)

