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Depletable Externalities and Pigouvian Taxation[1]

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In their book Baumol and Oates ["The Theory of Environmental Policy: Externalities, Public Outlays, and the Quality of Life," Prentice-Hall, Englewood Cliffs, N.J. (1975).] argue that whether an externality is depletable (private) or undepletable (public) is the key characteristic in determining the optimal pricing pattern. They argue that unlike the undepletable case a negative depletable externality requires not only a charge or tax on the generator of the externality but a payment or compensation to the victim in order to achieve Pareto optimality. It is shown that the key characteristic determining whether compensation of victims is required for efficiency is not the depletability of the externality but whether the victim can costlessly control or limit the amount of the damaging substance received.

I.

One of the major themes in Baumol and Oates' [1] excellent treatment of the theory of externalities is the distinction between depletable and undepletable externalities and the implications of this distinction for efficient pricing or Pigouvian taxation. The importance that Baumol and Oates attach to this distinction has been commented on by several of the reviewers of this book, for example, Collard [2], Crocker [4], Morgenstern [5], and Randall [6].

An undepletable externality is in the nature of a public good or public bad. Baumol and Oates (B & O hereafter) define an undepletable (public) externality as one "in which the amount of the externality consumed by one individual does not reduce the amount available to any other person or firm" [1, p. 35]. In contrast, for B & O, "The distinguishing characteristic of the depletable [private] externalities case is the reduction in the amount of the external product that remains for everyone else whenever some individual or firm increases its consumption of that item" [1, p. 46, word in brackets added]. They show that in the case of undepletable externalities, optimal resource allocation calls for a per unit tax (subsidy) on the generator of a negative (positive) externality but a zero compensation (price) to those affected by it. But they argue that in the case of a depletable externality efficient resource allocation calls for a single price per unit of the externality paid by (to) the generator of a negative (positive) externality and paid to (by) its recipient.

Unfortunately this conclusion is not correct. This is because although the internal logic of their model is correct, it is not a model of an externality as they have defined

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the term. In this note, I develop a model of the optimal taxation of a depletable externality which is consistent with B & O's definitions. I show that there is no fundamental difference in the properties of optimal tax/price schemes for depletable and undepletable externalities. The difference in tax/price prescriptions that B & O find arises because in their model receptors are free to choose the quantity of the "externality" they receive. But this freedom is not consistent with their definition of an externality. B & O define an externality as follows:

> An externality is present whenever some individual's (say A's) utility or production relationships include real (that is, nonmonetary) variables, *whose values are chosen by others* (persons, corporations, governments) without particular attention to the effects on A's welfare." [1, p. 17, original italics omitted, emphasis added by the author]

As B & O say, definitions are a matter of taste and convenience. But they have persuaded me that their definition is to be preferred to others they discuss on the grounds that it focuses on the technical or physical aspects of the interconnections among economic agents and leaves appropriate institutional responses and policy recommendations to be deduced from the definition.

B & O reach their erroneous conclusion regarding taxation of depletable externalities because they apparently overlooked a key feature of their definition of an externality—that the recipient has no control over the amount received.[2] To derive the properties of the optimal price structure, B & O differentiate the constrained expenditure and profit functions of individuals and firms with respect to their choice variables. And since B & O take derivatives with respect to the quantities of the externality received, they are in effect assuming that individuals have some choice over these quantities. In fact in their model the role of the prices paid to recipients of negative externalities is to induce recipients to accept the optimal quantities. And recipients must be charged for a beneficial externality to prevent them from trying to take too much. The prices are necessary to get the "market" to clear.[3] In B & O's model the prices arise so as to regulate recipients' choices, not because of the absence of choice inherent in an externality. If individuals cannot choose the quantities of the externality they receive, there is no need to include a price in their constraint set.

II.

I will now derive the efficient or Pareto optimal prices and taxes for a negative depletable externality, that is, a variable produced by firms which reduces individual utilities, the quantities of which are chosen by profit-maximizing firms, and for which one individual's absorption of a given quantity (over which he has no control) reduces the quantity to be absorbed by others.[4] B & O express some doubt as to

[2] It is important to distinguish between costless control of the amount received and costly control (or influence) over the effects of the externality. For example, people may be able to reduce the loss of utility caused by exposure to air pollution through costly mitigating or averting activities. These activities affect the magnitude of marginal damages and therefore the amount of the optimal price charged to the generators of the externality. On this see Courant and Porter [3]. But as shown below, the possibility of mitigating activities does not alter the conclusions regarding prices to externality recipients.

[3] B & O may have sensed the nature of the problem and the real reason why their two models yielded different conclusions about prices. See p. 18, note 13, and p. 23. But they explicitly maintained their original definition of an externality, nevertheless.

[4] B & O treat the more general case in which both individuals and firms are affected by the externality.

whether there are any real cases of negative depletable externalities. Acid deposition on land seems to fit the definition. Each pound of sulfur emitted to the atmosphere must land somewhere, and if the quantity falling on A's land increases, there is less to fall elsewhere. The quantity of acid deposition falling on a parcel of land of given size depends on the total emissions of precursors and on the pattern of atmospheric transformation and transport. No recipient can influence the extent to which he "depletes" the externality. Thus depletability is irrelevant for optimal pricing.

Following B & O's notation for the most part:

x_{ij} = the amount of good (resource) i consumed by individual j, $(i = 1,\dots,n)(j = 1,\dots,m)$,

y_{ik} = the amount of good (resource) i produced (used) by firm k, $(i = 1,\dots,n)(k = 1,\dots,h)$,

r_i = the total quantity of resource i available to the economy,

s_k = the quantity of the externality produced by firm k, measured in physical units, e.g., tons of sulfur,

s_j = the quantity of the externality absorbed by individual j, also measured in physical units,

and $\Sigma_j s_j = S = \Sigma_k s_k$.

The utility and production relations are

$$u^j(x_{1j},\dots,x_{nj},s_j)$$

and

$$f^k(y_{1k},\dots,y_{nk},s_k) = 0.$$

The conditions for a Pareto optimum can be found by maximizing $u^1(x_{11},\dots,x_{n1},s_1)$ subject to:

$$u^j(x_{1j},\dots,x_{nj},s_j) = u^{*j} \qquad (j = 2,\dots,m) \tag{1}$$

$$f^k(y_{1k},\dots,y_{nk},s_k) = 0 \qquad (k = 1,\dots,h) \tag{2}$$

$$r_i + \sum_{k=1}^{h} y_{ik} - \sum_{j=1}^{m} x_{ij} = 0 \qquad (i = 1,\dots,n) \tag{3}$$

and

$$\sum_{j=1}^{m} s_j = \sum_{k=1}^{h} s_k = S \tag{4}$$

where λ_j, μ_k, and ω_i will be the multipliers on the constraints (1), (2), and (3).

To solve the problem, we must specify how the total emissions are distributed among individuals. For the moment, let us assume that emissions from all sources mix in the atmosphere and are then distributed so that each individual receives $\alpha_j S$ where the α_j are given by some atmospheric dispersion process and $\Sigma_{j=1}^{m}\alpha_j = 1$. Constraint (4) can now be entered in (1) since[5] $s_j = \alpha_j \Sigma_{k=1}^{h} s_k$.

The general principles can be seen through the analysis of the simpler case in which only individuals are affected.

[5]This would be the case, for example, if all firms were located at the same point in space.

The Pareto optimum conditions are[6]

$$\lambda_j u_i^j = \omega_i \qquad \text{(all } i, j) \tag{5}$$

$$-\mu_k f_i^k = \omega_i \qquad \text{(all } i, k) \tag{6}$$

$$\sum_{j=1}^{m} \alpha_j \lambda_j u_{s_j}^j = -\mu_k f_{s_k}^k \qquad \text{(all } k).^7 \tag{7}$$

Condition (7) is of greatest interest. It requires that each firm's marginal cost of reducing emissions equal the aggregate marginal damage that a unit of emission causes as it is dispersed across the population. Since the s_j are determined given $\Sigma_k s_k$, there is no condition governing the optimum distribution of S across the population. In this respect the undepletable and depletable cases are essentially similar. It should be no surprise that the optimal price/tax schemes are also the same.

Following B & O, we write expenditure minimization and profit maximization expressions for individuals and firms, including experimentally prices on s_k and s_j, and solve them to find the prices that make the maximization and minimization conditions correspond to the Pareto optimum conditions. Individual j wishes to minimize

$$\sum_{i=1}^{n} p_i x_{ij} + p_{s_j} s_j + \gamma_j \left[u^{*j} - u^j(\cdot) \right].$$

But since s_j is not a choice variable for the individual, p_{s_j} plays no role in achieving Pareto optimality. Firm k wishes to maximize

$$\sum_{i}^{n} p_i y_{ik} - t_k s_k + \beta_k f^k(\cdot).$$

This calls for:

$$p_i = -\beta_k f_i^k \qquad \text{(all } i)$$

and

$$t_k = \beta_k f_{s_k}^k.$$

Comparing these equations with (6) and (7) shows that the market will reach Pareto optimality if:

$$P_i = \omega_i \qquad \text{(all } i)$$

and

$$t_k = \sum_{j=1}^{m} \alpha_j \lambda_j u_{s_j}^j. \tag{8}$$

[6]For notational simplicity, I assume an interior solution.

[7]Subscripts on u^j and f^k denote partial derivatives with respect to the indicated variables. Note also that μ_k, $f_{s_k}^k$, and $u_{s_j}^j$ are all negative.

Firms should be charged for their outputs of the negative externality, but the price to recipients is irrelevant to Pareto optimality.[8]

III.

I now consider several modifications and extensions of the basic model of the preceding section. The first extension is to recognize that there is a sense in which individuals do have some control over the extent to which a negative externality affects their utility level. They might be able to place insulation in their homes to reduce the impact of aircraft or traffic noise; they might be able to counter the possible adverse effect of acid rain by applying fertilizer or lime to their gradens; or they might be able to reduce directly their inhalation of air pollutants by installing air conditioning and filtration equipment. Of course these mitigating activities are themselves costly; and even if they protect perfectly against the externality, there is a loss of utility due to the diversion of resources away from other consumption.

The possibility of mitigating behavior can be accommodated in the specification of the utility function. First, suppose the utility function is strongly separable in s_k. The $u^j_{s_j}$ would be independent of x^j_i. The individual could not alter the marginal damage caused by s_j by any change in the levels of goods consumed. Mitigating behavior would not be possible. But if the utility function were not separable in s_j, then the marginal damage of s_j depends on x^j_i. The individual can reduce the marginal damage by increasing his consumption of some mitigating goods (and because of the budget constraint reducing his consumption of some other goods). These possibilities were implicit in the preceding model and can be made explicit by writing the relevant Pareto optimum condition (7) and optimal Pigouvian tax (8) as:

$$\sum_{j=1}^{m} \alpha_j \lambda_j u^j_{s_j}\left(x^j_i; s_j\right) = -\mu_k f^k_{s_k} \tag{7'}$$

$$t_k = \sum_{j=1}^{m} \alpha_j \lambda_j u^j_{s_j}\left(x^j_i; s_j\right). \tag{8'}$$

If individuals can choose the extent to which a given negative externality will damage them through mitigating behavior, then these possibilities must be taken into account in measuring marginal damages and establishing the optimal tax on s_k.[9] But this possibility is not related to the distinction between depletable and undepletable externalities. And it does not alter our conclusion that compensation for damages is unnecessary for Pareto optimality.

Now let us relax the implicit assumption of perfect mixing of emissions and assume that it is possible to trace each unit of emission from its source k to its receptor j. For simplicity let the process be described by:

$$s_{jk} = \alpha_{jk} s_k, \qquad \sum_{j=1}^{m} \alpha_{jk} = 1$$

[8] This is more accurate than saying that the optimal price to recipients is zero. Any positive or negative p_{s_j} creates a form of nondistorting lump sum tax or subsidy with no efficiency implications.

[9] On this possibility see, for example, Courant and Porter [3].

where s_{jk} is the quantity of s received by j which can be attributed to source k. Thus:

$$s_j = \sum_{k=1}^{h} s_{jk} = \sum_{k=1}^{h} \alpha_{jk} s_k.$$

Substituting this in (1) and solving for the Pareto optimum conditions yields among others:

$$\sum_{k=1}^{h} \alpha_{jk} s_k \lambda_j u^j_{s_j} = \mu_k k^k_{s_k} \qquad (\text{all } k, j). \tag{7''}$$

Since the Pareto optimum conditions involve a set of bilateral relationships between every possible source–receptor combination, it might be possible to define property rights and allow bilateral exchanges to be the means of achieving an optimum. Aside from large numbers and likely high transactions costs, the principal barrier to a market solution is the nonseparability of the left-hand side of (7″). That is, unless $u^j_{s_k}$ is constant, the price that k would pay j (or be paid by j) depends not only on k's emission, but also on the total of all other emissions reaching j.

If all bargains are precluded by transactions costs or by regulation, the pricing solution is clear. Each source should be charged a unit tax given by the left-hand side of (7″). With differences among receptors and different α_{jk}'s, each source's price is likely to be different. But again since individuals cannot choose the quantities to be received from any sources, prices to individuals have no role to play in achieving Pareto optimality.

Finally assume that each source can control the destination of its emissions, i.e., that firm k can choose its α_{jk} subject to $\sum_{j=1}^{m} \alpha_{jk} = 1$. In practice a firm might be able to choose whether to emit a substance to the air, scrub it from its exhaust gas, and discharge the substance into a river, or deposit it as a sludge in a landfill site. If firm k can affect the distribution of s_k among individuals through its choice of discharge media, location or height of smokestacks, etc., then it should be faced with a set of differential taxes that reflect the differences in marginal damages caused by its choice of where and how to discharge the substance. But the general nature of the pricing rules is not affected by this complexity.

IV.

In summary, the principal conclusion to be drawn from this analysis is that the distinction between depletable and undepletable externalities is unimportant, at least as far as the basic features of the optimal Pigouvian tax policy are concerned. The asymmetry between the price paid by the source and the price paid to the receptor that B & O found in the case of undepletable externalities is also present in the case of depletable externalities, at least when both types of externalities are analyzed in a common framework with a consistent set of definitions. And since the essence of the externality problem is that individuals cannot choose the level of the externality affecting them, prices on or compensation for the externality received have no role to play in directing the recipients of externalities toward a Pareto optimal or efficient allocation.

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