A general equilibrium analysis of the optimal number of firms in a polluting industry

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Abstract. In partial equilibrium competitive models in which emissions are proportional to output, the optimal output of individual firms in a polluting industry is the same with or without a Pigouvian tax. The same holds in general equilibrium only if the production function of those firms is homothetic. The Pigouvian tax increases the price of the polluting good, reducing industry output, and, as a consequence, the relative price of the input in which polluting firms are relatively intensive. They will adjust their input ratios accordingly, and if production functions are non-homothetic, this will either increase or decrease the level of output at which the total profit of the individual firm is zero.

Une analyse en équilibre général du nombre optimal de firmes dans une industrie polluante. Dans les modèles concurrentiels en équilibre partiel, la production optimale des firmes individuelles dans une industrie polluante est identique qu'il y ait ou non une taxe à la Pigou. On obtient le même résultat en équilibre général si la fonction de production de ces firmes est homothétique. La taxe à la Pigou augmente le prix du produit polluant, réduit le niveau de production de l'industrie, et par voie de conséquence le prix relatif de l'intrant employé de façon relativement intensive par les firmes polluantes. Ces firmes vont ajuster les ratios d'intrants en conséquence. Si les fonctions de production ne sont pas homothétiques, ces ajustements vont augmenter ou réduire le niveau de production qui correspond à un profit total nul pour la firme individuelle.

INTRODUCTION

That a Pigouvian tax on emissions would foster the optimal number of firms in a polluting industry has been demonstrated with partial equilibrium analysis (see, e.g., Baumol and Oates, 1975; Burrows, 1979; and Schulze and D'Arge, 1974). In all these models each firm in the industry uses the same production function, which exhibits increasing and then, at an output that is small relative to the size of the market, decreasing returns to scale. There is a further simplification that emissions per unit of output of the polluting firms are invariant to changes in the ratio of their factors of production and also an implicit assumption (see Mestelman, 1981) that the

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polluting industry is a constant-cost industry. One of the interesting results obtained with these models is that the optimal size of the individual polluting firm is the same with or without the Pigouvian tax on emissions.

In a general equilibrium model in which the relative prices of inputs are sensitive to the quantities employed by the polluting industry, and total emissions of each firm are proportional to its output, the optimal level of output of the individual firm is the same with or without the Pigouvian tax only if its production function is homothetic. If the production function of the firm is non-homothetic, then the optimal size of the individual firm may be either greater or smaller after a Pigouvian tax is imposed.

In this paper, a simple general equilibrium model is used to demonstrate that the Pigouvian tax on pollution promotes an efficient allocation of inputs, including an efficient number of firms in each industry. There are reasons noted in the economic literature that pollution taxes might be incorrectly set, with the result that there would be too few or too many firms in the industry. Some of these possibilities are considered here. It is then explained, in the case of firms using non-homothetic production functions, why the long-run equilibrium levels of output of these individual firms could be different before and after the imposition of a Pigouvian tax. Finally, the general results are illustrated with a numerical example.

THE CONDITIONS FOR LONG-RUN EFFICIENCY

Consider an economy in which two goods, x and y, are produced by perfectly competitive industries, each consisting of a variable and divisible number of firms, n and m, respectively. The economy is situated in an airshed in which a pollutant is uniformly mixed. The production of good y causes pollution that reduces the productivity of firms making good x. The industry outputs are

$$x = n\,X(L_x, K_x, e), \tag{1}$$

$$y = m\,Y(L_y, K_y), \tag{2}$$

$$e = Ey, \tag{3}$$

where L_i and K_i are the quantities of labour and capital, respectively, used by the individual firm in the ith industry. The variable, e, is the pollutant concentration in the airshed, and the constant, E, is the emission factor per unit of good y produced. This constant measures the resulting contribution to the ambient air concentration of the pollutant, as in Kohn (1978). In this model, $X(L_x, K_x, e)$ is the common production function of firms in industry x and $Y(L_y, K_y)$ is the common production function of firms in industry y. Both production functions exhibit increasing and then decreasing returns to scale. (For convenience, lower-case letters pertain to industries and upper-case letters to firms.) It is assumed that the adverse effect of pollution depends on the quantity of y alone and is invariant to the ratio of factors in either industry.

Preferences of households for the two goods are represented by a community utility function, $U(x, y)$, and the total quantities of labour and capital are L_0 and K_0, respectively. All the first derivatives of the functions are positive except for X_e which is negative.

The necessary conditions for an efficient allocation of resources are derived from the Lagrangian expression,

$$\mathcal{L} = U(x,y) + \lambda(L_0 - nL_x - mL_y) + \gamma(K_0 - nK_x - mK_y), \tag{4}$$

and the relationships defined by equations (1), (2), and (3). Setting the derivatives of the Lagrangian equal to zero, in order to solve for an internal solution, yields the following:

$$L_x\colon\ U_x nX_L - n\lambda = 0 \tag{5}$$

$$K_x\colon\ U_x nX_K - n\gamma = 0 \tag{6}$$

$$L_y\colon\ U_x nX_e EmY_L + \mathrm{U_y mY_L} - m\lambda = 0 \tag{7}$$

$$K_y\colon\ U_x nX_e EmY_K + U_y mY_K - m\gamma = 0 \tag{8}$$

$$n\colon\ U_x X - \lambda L_x - \gamma K_x = 0 \tag{9}$$

$$m\colon\ U_x nX_e EY + U_y Y - \lambda L_y - \gamma K_y = 0, \tag{10}$$

where U_i, X_i, and Y_i are derivatives of that function with respect to the ith variable. Solving (5) and (7) simultaneously yields

$$U_x/U_y = Y_L/(X_L - nX_e EY_L) \tag{11}$$

This is the condition for the equality of the marginal rate of substitution and the marginal rate of transformation. The interpretation of the latter is as follows. If one unit of labour is transferred away from industry y, the output of that industry will decline by the marginal product of labour, Y_L. The output of industry x will increase by the marginal product, X_L, plus the increase in output of each firm in that industry as a consequence of the reduction in emissions associated with the marginal unit of good y, which is nX_eE, times the actual change in the output of industry y, which is Y_L. Because X_e is negative, the second term in the denominator of the marginal rate of transformation augments the first term.

From equations (5), (6), (7), and (8) it follows that

$$Y_L/Y_K = X_L/X_K, \tag{12}$$

which is the usual condition for equality of the marginal rates of technical substitution; it holds here because emissions are invariant to the ratio of inputs used in either industry.

Finally it follows from (5), (6), and (9) that

$$L_x X_L + K_x X_K = X. \tag{13}$$

This is the condition that each firm in the pollution sensitive industry operates with a combination of inputs such that Euler's theorem is satisfied; that is, the sum of the quantity of each input times its marginal product exhausts the total product. From equations (7), (8), and (10), it follows that the same applies to firms in the polluting industry:

$$L_y Y_L + K_y Y_K = Y. \tag{14}$$

This condition holds because, in the words of Schulze and d'Arge (1974, 766), 'in spite of the externality, social product is still maximized by producing each unit of (the polluting good) as cheaply as possible.'

THE COMPETITIVE MARKET EQUILIBRIUM WITH THE PIGOUVIAN TAX

The above marginal conditions are achieved in a perfectly competitive market economy in which firms produce quantities of output at which price equals marginal cost and in which there is a Pigouvian tax equal to the marginal damage per unit of pollution, imposed on emissions. It is assumed that the tax revenue is distributed to households and not as compensation to pollution-sensitive firms. Accordingly, the price of good x,which is p_x, is simply the price of either input divided by the marginal product of that input. If, for example, the wage is w, then

$$p_x = w/X_L. \tag{15}$$

The price of good y, p_y, is the sum of the private marginal cost, w/Y_L, plus the Pigouvian tax, ϕ, multiplied by the emission factor, E. This tax is equal to the reduction in the output of industry x caused by the marginal unit of pollution, which is $n(X_e)$, times the price of good x.

The Pigouvian tax and the price of good y are, respectively,

$$\phi = nX_e(w/X_L) \tag{16}$$

$$p_y = w/Y_L - \phi E. \tag{17}$$

In this market economy consumers will maximize their utility subject to their budget constraints, so that

$$U_x/U_y = \mathrm{p_x}/p_y \tag{18}$$

Substituting (15), (16), and (17) into (18) yields

$$U_x/U_y = Y_L/(X_L - nX_eEY_L), \tag{19}$$

which is identical to the optimal condition, (11), above. The Euler conditions, (13) and (14), for long-run efficiency are no different for an economy with or without pollution taxes and obtain as a consequence of cost minimization by firms and the free entry and exit of firms in each industry.

If the pollution tax exceeds the marginal damage per pound of pollution there will be too few firms in the polluting industry. That possibility is associated in the economic literature (see Rose-Ackerman, 1973, 518; and Burrows, 1979) with the case in which marginal damages increase with emissions, and (a) the government sets the tax rate so that the total tax revenue exactly compensates for total damage (in which case the tax exceeds marginal damage), or (b) the government sets the tax equal to initial marginal damage and then neglects to lower the tax rate as emissions, and hence marginal damages, predictably decline.

Alternatively, if the tax is less than the marginal damage, there will be too many firms in the polluting industry. This could occur if the pollutant tax is implemented incrementally and there is a non-convexity in the production technology. This can result in a competitive equilibrium that is inferior to the global optimum. Such an iterative sequence is modelled in Kohn and Aucamp (1976, 950–1). In either case, the correct Pigouvian tax is equal to marginal pollutant damage *after* the economy has adjusted to its optimal state.

It is generally accepted by economists that a Pigouvian tax based on marginal damages at the optimum level of emissions will, in the context of a perfectly competitive market economy, foster economic efficiency in both the short run and the long run. A counter-argument has been made by Carlton and Loury (1980) that a single tax on emissions will not foster an optimal number of polluting firms if the rate of emissions is a function of scale as well as output. However, Collinge and Oates (1982, 347n) argue that the result obtained by Carlton and Loury 'involves a misspecification of the Pigouvian tax.'

There is an aspect of the long-run efficiency of the Pigouvian tax that needs to be clarified. In the partial equilibrium models it is usually claimed that 'the firm's cost minimizing output and emissions levels will remain completely unaffected by the tax' (Baumol and Oates, (1975, 181)) although there is a special case conceived by Mestelman (1981, 127–8) in which the output of each firm in an increasing cost, polluting industry declines when the tax is imposed. These conclusions do not necessarily hold in the context of a general equilibrium model. The Pigouvian tax increases the relative price of the polluting industry, thereby decreasing the quantity demanded of that industry's output. As a result, there is likely to be a reduction in the relative price of the input in which the polluting industry is comparatively intense. If the production function, $Y = Y(L_Y, K_Y)$, is homothetic, the change in relative input prices will not affect the optimum level of output of individual firms in that industry. If the production function is non-homothetic, the change in relative prices will either increase or decrease the optimal level of output.

This can be demonstrated with the following well-known production function, which is used in Kohn (1983, 1984):

$$Y = AL^{\alpha}K^{\beta} - L^{\alpha+\delta}K^{\beta+\delta}, \tag{20}$$

in which $\alpha + \beta > 1$, $\delta > 0$, and $|\alpha - \beta| < 1$. The firm operating with this production function experiences increasing and then decreasing returns to scale. This function has the nice property that if $\alpha = \beta$, it is homothetic and non-homothetic otherwise. The Euler condition, which is (14) above, for this production function is

$$LK = [A(\alpha + \beta - 1)/(\alpha + \beta + 2\delta - 1)]^{1/\delta}. \tag{21}$$

Substituting this value for LK into (20), to determine the long-run equilibrium level of output, Y^*, of the individual firm, yields

$$Y^* = (K^{\beta-\alpha})\,(A[\alpha + \beta - 1]/[\alpha + \beta + 2\delta - 1])^{\alpha/\delta} \cdot (2A\delta/[\alpha + \beta + 2\delta - 1]). \tag{22}$$

If the production function is homothetic and $\alpha = \beta$, then K drops out and Y^* is a constant, independent of the capital-to-labour ratio. If the function is non-homothetic, $\alpha \neq \beta$, then the optimal level of output of the polluting firm (and therefore, the firm's total emissions as well) depends upon the cost-minimizing ratio of capital-to-labour. In this particular function, the level of output, Y^*, increases with the capital to labour ratio if $\beta > \alpha$ and decreases if $\alpha > \beta$. These results are illustrated in the following numerical example.

A NUMERICAL EXAMPLE OF THE NON-HOMOTHETIC CASE

The long-run efficiency of the Pigouvian tax in the case of non-homothetic production functions can be demonstrated with a numerical example. Let the common production function for firms in industry y be

$$Y = (48\, L_y^{2/3}\, K_y^{4/3} - L_y^{5/3}\, K_y^{7/3}). \tag{23}$$

There are m firms in this industry and the total output is $y = mY$. Let the production function for firms in industry x be

$$X = (12\, \mathrm{L}_{\mathrm{x}}^{4/3}\, K_x^{2/3} - L_x^{7/3}\, K_x^{5/3})(1 - e^2). \tag{24}$$

The emission factor, E, equals $1/[10{,}240\,(10^{1/2})]$ and the pollutant concentration in the airshed is therefore

$$e = y/[10{,}240\,(10^{1/2})]. \tag{25}$$

Total emissions in equation (24) are squared to model the case that concerned Rose-Ackerman (1973) and Burrows (1979), in which marginal damage increases with emissions. The correct Pigouvian fee in this model is $(n2X)\,(p_x)(e/(1 - e^2))$ per unit of emissions, E.

Assume that the preferences of consumers are represented by the utility function,

$$U = 465\, ln\, x + 692\, ln\, y, \tag{26}$$

where $x = nX$, and, finally, that the total quantities of resources are 29,696 units of labour and 80 units of capital. Setting the wage rate equal to unity, the competitive equilibrium allocations with and without a Pigouvian tax are shown in table 1.

As a consequence of the Pigouvian tax on emissions, the relative price of good x declines. Consumers then maximize utility by consuming more of good x and less of good y. Between the two equilibrium allocations, the level of utility associated with the Pigouvian tax is the higher. The increase in the output of good x is accomplished because more of the economy's inputs are allocated to that industry and also because the pollution level, and therefore the loss of output caused by pollution, is reduced.

TABLE 1
Competitive equilibrium allocations with and without a Pigouvian tax: solution of the numerical example

	(1) Without a Pigouvian tax[a]	(2) With a Pigouvian tax [a]
(ϕ) Pigouvian tax on emissions	-	$41,783-
(p_x) Price of good x	$0.43-	$0.34+
(p_y) Price of good y	$2.11+	$2.79+
(p_x/p_y) Relative price of good x	0.20+	0.12+
(x) Output of industry x	64,325+	89,107+
(y) Output of industry y	19,335+	16,384
(U) Community utility level	11,978+	12,015+
(ℓ_y) Labour employed in industry x	22,885−	25,600
(k_x) Capital employed in industry x	9.5−	16
(ℓ_y) Labour employed in industry y	6,811+	4,096
(k_y) Capital employed in industry y	70.5+	64
(e) Pollution level	0.6−	0.5+
(r) Price of capital	$482.9+	$320
(r/w) Relative price of capital	482.9+	320
(X) Output of each firm in industry x	276.2+	278.5−
(Y) Output of each firm in industry y	111.6−	128
(n) Number of firms in industry x	233−	320
(m) Number of firms in industry y	173+	128
(L_x) Quantity of labour employed by each firm in industry x	98.3−	80
(K_x) Quantity of capital employed by each firm in industry x	0.04+	0.05
(L_x/K_x) Labour to capital ratio in industry x	2415−	1600
(L_y) Quantity of labour employed by each firm in industry y	39.3+	32
(K_y) Quantity of capital employed by each firm in industry y	0.41−	0.5
(L_y/K_y) Labour to capital ratio in industry y	96.6−	64
($P_x x + p_y y$) Gross National Product	$68,329+	$76,437−
($w\ell + rk$) Factor income	$68,329+	$55,296
(ϕe) Pigouvian tax revenue	-	$21,141−
($p_x X - wL_x - rk_x$) Total profit of each firm in industry x	0	0
($p_y Y - wL_y - rK_y - \phi E$) Total profit of each firm in industry y	0	0

[a]All entries followed by a plus (minus) sign are rounded down (up).

The decline in the output of the polluting good reflects only the reduction in inputs employed by that industry.

By comparison, industry x is labour intensive and industry y is capital intensive. This is readily observed from the industry production functions, which are

$$x = (16)l_x^{5/6}k_x^{1/6})(1 - y^2/1,048,576,000) \tag{27}$$

$$y = 128l_y^{1/6}k_y^{5/6}. \tag{28}$$

As a consequence of the expansion of the first industry and the contraction of the second, the relative price of capital declines (The wage rate in this model is arbitrarily fixed at $1.00) and firms in both industries use less labour relative to capital. This is possible in an economy with fixed supplies of inputs only because the expanding industry is labour intensive. (The analysis here is similar to that in Boadway, 1979, 299–306.)

Firms in both industries have non-homothetic production functions. As the relative price of capital declines, the quantity of output at which Euler's theorem is satisfied decreases for firms in industry x and increases for firms in industry y. This is evident in table 1 for industry y and also for industry x when the adverse effect of pollution on that industry is taken into account. The final entries in the table confirm that there is long-run equilibrium with zero profits in both competitive equilibrium allocations.

It may be observed that this numerical example has properties characteristic of a well-known model in Baumol (1972, 317) in which the production possibility frontier because concave to the origin as it approaches the axis of the pollution-sensitive industry and convex to the origin as it approaches the axis of the polluting industry. In this example, the inflection point occurs when $x \simeq 52{,}073$ and $y \simeq 20{,}744$. This combination is above the initial competitive equilibrium in table 1. Accordingly, the case noted earlier in this paper, in which an iterative sequence of pollution taxes (assessed in the absence of advance knowledge of the correct tax) could bring the economy to an equilibrium in which there are too many firms in the polluting industry, would not occur in this particular model.

REFERENCES

Baumol, William J. (1982) "On taxation and the control of externalities." *American Economic Review* 62, 307–22

Baumol, William J. and Wallace E. Oates (1975) *The Theory of Environmental Policy* (Englewood Cliffs, NJ: Prentice Hall)

Boadway, Robin W. (1979) *Public Sector Economics* (Boston: Little, Brown and Co.)

Burrows, Paul (1979) 'Pigouvian taxes, polluter subsidies, regulation, and the size of a polluting industry.' This JOURNAL 12, 494–501

Carlton, Dennis W. and Glenn C. Loury (1980) 'The limitations of Pigouvian taxes as a long-run remedy for externalities.' *Quarterly Journal of Economics* 95, 559–66

Collinge, Robert A. and Wallace E. Oates (1982) 'Efficiency in pollution control in the short and long runs: a system of rental emission permits.' This JOURNAL 15, 346–54.

Kohn, Robert E. (1978) *A Linear Programming Model for Air Pollution Control* (Cambridge, MA: MIT), 113–14

—(1983) 'Returns to Scale in Welfare Economics,' *Atlantic Economic Journal*, 11, 91–95

—(1984) 'Industry isoquants when the number of firms is a variable.' *Rivista Internazionale di Scienze e Commerciali*, 31, 257–63

Kohn, Robert E. and Donald C. Aucamp (1976) 'Abatement, avoidance, and nonconvexity.' *American Economic Review* 66, 947–52

Mestelman, Stuart (1981) 'Corrective production subsidies in an increasing cost industry: a note on a Baumol-Oates proposition.' This JOURNAL 14, 124–30

Rose-Ackerman, Susan (1973) 'Effluent charges: a critique.' This JOURNAL 6, 512–28

Schulze, William and Ralph C. d'Arge (1974) 'The Coase proposition, information constraints, and long-run equilibrium.' *American Economic Review* 64, 763–72