The rate of emission and the optimal scale of the polluting firm

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Abstract. Empirical evidence suggests that in some cases, average emissions may decrease and then increase with the polluting firm's level of output. If the U-shaped average emission curve bottoms out at the same output at which the U-shaped average cost of production curve bottoms out, the efficient scale of the polluting firm is the same with a Pigouvian tax as without the tax. In such a case total emissions of the polluting industry are proportional to the total output of the industry, and the results derived in Kohn (1985) for a constant rate of emissions hold.

Le taux d'émission de matières polluantes et la taille optimale de la firme polluante. Des résultats empiriques suggèrent que dans certains cas, le degré moyen de pollution peut décroître puis s'accroître à proportion que le niveau de production de la firme polluante s'accroît. Si le degré de pollution moyen a un profil en forme de U qui a son minimum au même niveau de production que la courbe des coûts moyens de production (qui a elle aussi un profil en forme de U), la taille efficiente de la firme polluante est identique avec ou sans une taxe à la Pigou. Dans un tel cas, la quantité totale d'émission de matières polluantes est proportionnelle au niveau de production totale de l'industrie et les résultats dérivés dans les travaux de Kohn (1985), pour le cas particulier où le taux d'émission est constant, demeurent valides.

INTRODUCTION

In a recent paper in this JOURNAL (Kohn, 1985), it is demonstrated that a Pigouvian tax on emissions fosters the optimal number of firms in a polluting industry.[1] In that model it is assumed that the total emissions of the firm are

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1 There are three substantive errors in the 1985 paper that should be corrected. The assertion (Kohn, 1985, 349) that equation (13) satisfies Euler's theorem has given the false impression

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proportional to its output. Although the assumption of proportionality is consistent with preponderant empirical documentation of constant emission factors (See Kohn, 1978, 38–65), it has been suggested by Carlton and Loury (1980) and by Spulber (1985) that emissions per unit of output may in some cases decrease and then increase as output increases. This paper presents some empirical data that confirm their hypothesis and sets forth the condition for the efficient scale of a polluting firm whose rate of emission varies with output. Of special interest is the case, originally conceived by Carlton and Loury (1980), in which average emissions are a minimum at the same output at which average private cost is a minimum.

EMPIRICAL EVIDENCE OF THE RATE OF EMISSIONS

In the third and latest *Compilation of Air Pollutant Emission Factors* (1977, 1978), as in preceding editions, emissions from manufacturing processes are with few exceptions presented as being proportional to output, as, for example, pounds of pollutant per ton of glass, steel, or cement produced. In cases where a range of emission factors are given, this is typically explained by 'variations among furnace types, charge types, quality, extent of pretreatment (etc.)' (1978, 7.9–4) rather than by changes in the rate of output. Because emission factors are empirically determined, presumably by observing plants operating at an efficient size, it could be assumed that the relationship of emissions to varying rates of output is not being observed.

An interesting exception is a table of emission factors for diesel-powered generators in vessels (1977, 3.2.3–5), which charts the emission rate for four levels of electrical output, measured in kilowatts. The electric output (as well as diesel fuel input) is approximately proportional to the kilowatt rating of the generator, provided that the load factor is held constant. It is reasonable to treat changes in kilowatt output as a proxy for changes in scale, achieved by proportional increases in motor size and fuel input.

In figure 1, some of the more interesting relationships between emissions

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that the production functions are intended to be linearly homogeneous. According to Pearce (1983, 135) Varian (1984, 330), and others, Euler's theorem is satisfied if *any* combination of inputs times their respective marginal products exhausts the total product, which is true only if the production function is linearly homogeneous. All of the production functions in the 1985 paper and in this paper are non-homogeneous and the total product is exhausted only by those combinations of inputs corresponding to points of constant returns to scale. Second, the problem that Rose-Ackerman (1973, 514) posed does not occur because 'the government sets the tax rate so that the total tax revenue exactly compensates for total damage (Kohn, 1985, 351)' but occurs if the damage eliminated by the exit of a polluting firm is significantly less than nX_e times the total emissions of that firm. This problem, which relates to the relative size of the marginal firm, is clarified by Collinge and Oates (1982, 348) and by Kohn (1986a). Third, the tax that Carlton and Loury (1980) evaluate is not 'a single tax on emissions (Kohn, 1985, 351)' but is a single tax on output. (The choice of output as a basis for a Pigouvian tax is discussed in Spulber (1985, 109, 113) and Kohn (1986).)

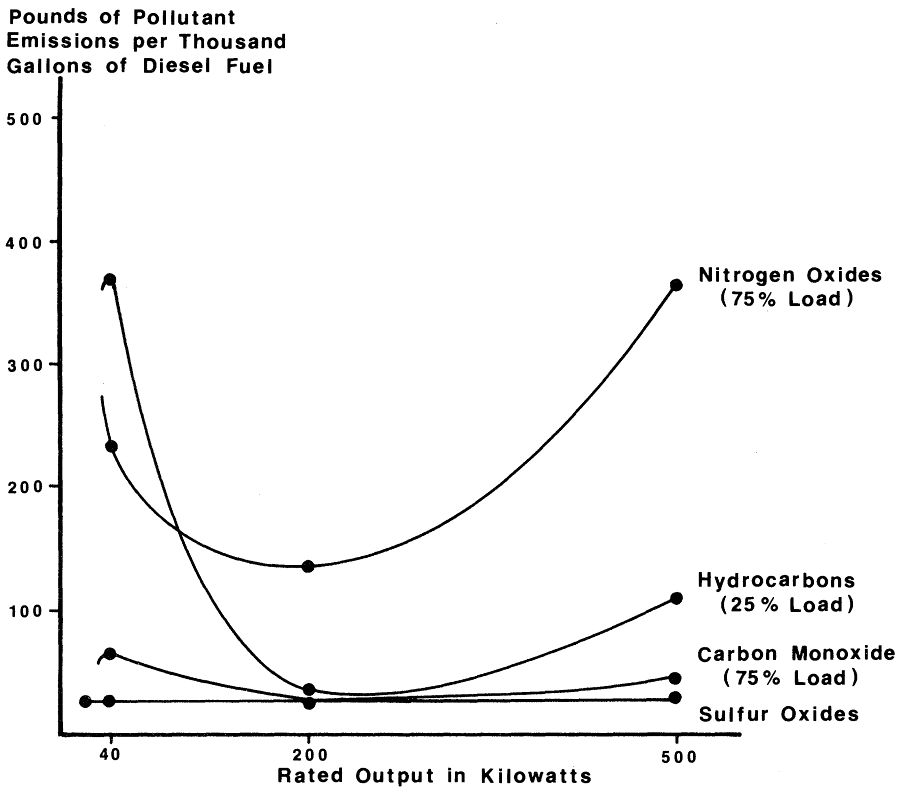

FIGURE 1

and kilowatt output are characterized by smooth curves arbitrarily drawn through each set of three points taken from the above mentioned table. Because the emission factors are total emissions divided by thousands of gallons of fuel, the curves in figure 1 are analogous to long-run average cost curves. With the exception of sulfur oxides, of which emissions are a constant twenty-seven pounds per thousand gallons of fuel, the U-shaped curves in figure 1 suggest the possibility that total emissions may, like conventional private costs, increase at a decreasing and then increasing rate with respect to output.[2]

EFFICIENT SCALE AND THE RATE OF EMISSIONS

In the previous paper (Kohn, 1985) the production sector of a general

2 For a rated output of twenty kilowatts, 204 pounds of hydrocarbons and twenty-nine pounds of carbon monoxide are emitted. (This explains what appear to be local maximums at the beginning of the corresponding curves in figure 1.) However, the twenty-kilowatt capacity represents what may be considered a different technology, that of small, portable type generators as opposed to the larger, stationary type.

equilibrium economy is represented by a polluting industry consisting of m firms using a common production function, $Y(L_y, K_y)$, and a pollution-sensitive industry, consisting of n firms using a common production function, $X(L_x, K_x, e)$. Both production functions are non-homogeneous and exhibit increasing and then decreasing returns to scale. There is a fixed quantity of labour, $mL_y + nL_x$, and a fixed quantity of capital, $mK_y + nK_x$, and the industry outputs are $y = mY$ and $x = nX$, respectively. In this paper the assumption that total emissions, e, are proportional to total output, y, is replaced by a more generalized function,

$$e = mE(Y)Y, \tag{1}$$

in which emissions per unit of output vary with output. The emission rate, E, may also be interpreted as average emissions per unit of Y. For simplicity, it is assumed that firms cannot alter the emission rate by expending inputs on abatement. The Lagrangian expression, with x and y replaced by their functional equivalents, is

$$\mathscr{L} = U\{nX[L_x, K_x, mE(Y)Y], mY(L_y, K_y)\} + \lambda(L_0 - nL_x - mL_y) + \gamma(K_0 - nK_x - mK_y). \tag{2}$$

In the revised model the first-order conditions for an internal solution include the same conditions for equal marginal rates of technical substitution between inputs and for product exhaustion by firms in industry x as before. The marginal rate of transformation is also the same, except that the rate of change of a polluting firm's emissions with respect to its output is now $(E_yY + E)$ instead of E. The more interesting revision is in the condition for the efficient scale of each polluting firm. In the preceding model that condition was

$$L_yY_L + K_yY_K = Y \tag{3}$$

which is the condition that the sum of inputs times respective marginal products exhausts the firm's total product. This condition, in the context of a perfectly competitive market economy in long-run equilibrium, has the following interpretation (Kohn, 1985). The imposition of a Pigouvian tax is likely to alter relative prices of outputs and hence, of inputs, the latter causing input ratios to change. If the emission rate is constant, the scale of polluting firms will not change if their production function is homothetic but will either decrease or increase if it is non-homothetic. Regardless of whether the production function is homothetic or non-homothetic, the polluting firm will always operate where the sum of private marginal products times the respective input quantities exhausts the total private product.

When the original assumption of constant emissions is replaced by equation (1) above, the new condition for the efficient scale of the polluting firm is

$$Y - L_yY_L - K_yY_K = nX_e(Y_L/X_L)E_yY^2, \tag{4}$$

where nX_e, which is negative, is the change in the output of industry x caused by the marginal unit of pollution and E_y, which can be negative, zero, or positive, is the change in the emission rate associated with the marginal unit of polluting output. This condition indicates whether the polluting firm operates in the region of increasing returns to scale ($Y < L_y Y_L + K_y Y_K$), constant returns to scale, or decreasing returns to scale ($Y > L_y Y_L + K_y Y_K$), according to the proportionate definition of returns to scale.

If E_y is negative at optimal Y, which holds when the emission rate decreases as output increases, the right-hand-side of (4) is positive, and it is efficient for the polluting firm to operate in the range of decreasing returns. If E_y is positive at Y, the rate of emissions is increasing and the firm should operate under increasing returns. If E_y is zero at Y, it is efficient for the firm to operate at the point of constant returns to scale. This is the case in Kohn (1985) in which the emission rate is a constant. It also holds in the case of a U-shaped emission curve when the efficient output is that at which E_y is zero.

Condition (4) can be interpreted as the condition for average-cost minimization by a firm in industry y when its costs include the Pigouvian tax on emissions, which is $nX_e(Y_L/X_L)$ measured in units of good y. It can also be interpreted as the condition for the efficient number of firms in the polluting industry; for at the margin and based on the efficient first derivatives, a one unit change in the number of firms, m, would have the same effect on output through the production effect as through the pollution effect. Consider, for example, the case in which E_y is positive at Y and polluting firms are operating under increasing returns to scale. If there were one less firm in the industry the forgone output of that firm would be Y units. But if the inputs formerly employed by that firm, L_y units of labour and K_y units of capital, were distributed equally among the remaining $(m - 1)$ firms in the industry, each firm would increase its output by $[L_y/(m - 1)]Y_L + [K_y/(m - 1)]Y_K$. Because firms are operating under increasing returns to scale, there is a net increase in industry output equal to $(m - 1)[L_y Y_L + K_y Y_K)/(m - 1)] - Y$. Turning now to the right-hand side of equation (4), the forgone emissions of the departing firm would be $E \cdot Y$ units, but each remaining firm would increase emissions by $[(E_y + E)/(m - 1)]Y$ for a net increase in total emissions of $(m - 1)[E_y Y^2/(m - 1)]Y$. This increase in total emissions multiplied by the Pigouvian tax, measured in units of good Y, is equivalent to a quantity of good Y that equals the left-hand side of the equation. Condition (4) thus balances the change in production associated with returns to scale and the marginal firm with the corresponding change in total emissions converted to the same production units.

A NUMERICAL EXAMPLE OF THE NON-HOMOTHETIC CASE

In Kohn (1985) a general equilibrium model is presented in which production and pollution damage are represented by the equations

$$x = nX = n(12L_x{}^{4/3}K_x{}^{2/3} - L_x{}^{7/3}K_x{}^{5/3})(1 - e^2) \quad (5)$$

$$y = mY = m(48L_y{}^{2/3}K_y{}^{4/3} - L_y{}^{5/3}K_y{}^{7/3}) \quad (6)$$

$$nL_x + mL_y = 29{,}696 \quad (7)$$

$$nK_x + mK_y = 80. \quad (8)$$

Equations (5) and (6) are a normalized, non-homothetic adaptation of a homothetic production function in Henderson and Quandt (1958, 46). In the new version of the model the emission rate is

$$E = (Y^2 - aY + b)/10^9 \quad (9)$$

in which average emissions are a decreasing and then an increasing function of Y. For $a = 256$ and $b = 20{,}000$, the emission rate is a minimum when $Y = 128$. In that case the efficient allocation, ($L_y = 32$, $K_y = 0.5$, $L_x = 80$, $K_x = 0.05$, $n = 320$, $m = 128$), is identical to the optimal allocation in the previous paper (Kohn, 1985, 353).

For simplicity, the utility function in the previous numerical example may be ignored, and the problem converted to one in which the quantity of good x is maximized subject to the constraint that y equal the same 16,384 as in the preceding solution. For $a = 200$ and $b = 20{,}000$, average emissions are a minimum at $Y = 100$, and the technically efficient solution is one in which the polluting firms operate in the range of increasing returns to scale. For $a = 400$, $b = 50{,}000$, and $y = 16{,}384$, the technically efficient solution is one in which the polluting firms operate in the range of decreasing returns to scale. The efficient quantities, together with the Pigouvian tax, are shown in table 1.

The general results in the table do not depend on the non-homotheticity of the polluters' production function. If the production function were homothetic, the dot product, $L_yY_L + K_yY_K$, would be 128 in all three columns, but the polluting firms would still operate under increasing returns or decreasing returns as in the non-homothetic case. The non-homotheticity does add interest to the results. In the case in which the polluting firms operate in the range of increasing returns to scale (see column 1), they are in fact cutting back production from the pre-tax output level of 128. Because the contracting firms are in the industry that is relatively capital intensive, the marginal rate of substitution of capital for labour in both industries declines. In this example the capital inputs in the production function for good y are higher-order terms than the labour inputs, and the increase in the labour-capital ratio therefore accentuates the decline in the efficient scale of the polluting firms.[3] In the case in which the polluting firms expand into the range of decreasing returns, those results are reversed.

3 This process of adjustment is described in Boadway (1979, 299–306). It is an interesting variant, however, in that total industry output is held constant and it is the individual firms in the industry that are contracting in scale, but increasing in number.

TABLE 1

Technically efficient allocations for three different emission functions[a]

Variable	(1) $E = (Y^2 - 200Y + 20{,}000)/10^9$	(2) $E = (Y^2 - 256Y + 20{,}000)/10^9$	(3) $E = (Y^2 - 400Y + 50{,}000)/10^9$
L_y	32.420	32.000	30.098
K_y	0.47825	0.50000	0.57264
Y	123.57	128.00	141.55
$L_y Y_L + K_y Y_K$	129.21	128.00	124.50
Emission tax in units of Y	−7848.7	−2435.2	−7279.1
$E_y Y^2$	0.00071965	0	−0.0023422
L_x	78.256	80.000	87.456
K_x	0.051111	0.05000	0.045777
n	324.54	320.00	299.72
m	132.59	128.00	115.744
$Y_K/Y_L = X_K/X_L$	306.24	320.00	381.77
e	0.17294	0.059245	0.21981
E_y	$47.133/10^9$	0	$-116.89/10^9$
Returns to scale	increasing	constant	decreasing

[a]All numbers are carried to five significant digits and rounded.

CONCLUDING REMARKS

In the case in which average emissions decline and then rise with output, the polluting firm may be conceived as having two U-shaped average cost curves, one for costs of production and the other for pollution damage costs. If the latter curve, which is simply average emissions times a constant Pigouvian fee, bottoms out first, the efficient output of the firm will be to the left of that at which its average production costs are a minimum. At that quantity, the emission rate is necessarily rising with output, and it is efficient for the firm to operate in the region of increasing returns to scale. If the U-shaped average pollution cost curve bottoms out after the average production cost curve, the reverse is true; average emissions are declining and it is efficient for the polluting firm to operate in the region of decreasing returns. This contrasts with the case in which the rate of emission is constant with respect to output and it is efficient for the polluting firm to operate at the scale at which the average cost of production is a minimum.

Empirical evidence suggests that average emissions may decline and then rise. Moreover, there is reason to presume that in some cases average emissions may be a minimum at the same quantity of output at which average production costs are a minimum.[4] In that special case there is a single efficient scale for the

4 This would follow, under suitable assumptions, from the 'materials-flows' model of externalities (see Burrows, 1980, 4), particularly with regard to emissions that are the 'natural resources utilized by the economy ... [and] thrown out by the production process itself.'

polluting firm, independent of the (non-zero) magnitude of the Pigouvian tax, and it follows that emissions are proportional to the output of the industry even though they are not proportional to the output of the individual firms in the industry. In that case the efficient scale of the polluting firm depends entirely on the homotheticity or non-homotheticity of the production function, and the analysis in Kohn (1985) is relevant, even though the emission rate is not constant with respect to the firm's output.

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