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Effluent Regulation and Long-Run Optimality

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The long-run efficiency properties of regulatory instruments are examined in a multiple-input framework. The effluent tax and tradeable permit are shown to be efficient with free entry and exit of small firms. The across-the-board effluent standard results in excessive entry and excessive industry pollution. © 1985 Academic Press, Inc.

1. INTRODUCTION

A regulator faced with free entry of firms is concerned not only with the implications of public policy for the discharges from a representative firm but also with the total pollution observed at the long-run market equilibrium. The regulator must take into account not only the effects of regulations upon the net marginal returns of the firm but must also consider the total profit of the firm and incentives for entry and exit. This paper shows that effluent charges and tradeable effluent permits lead to long-run optimality with entry of small firms. The difference between firm payments and environmental damages is interpreted as the rent accruing to the environmental resource. This result applies only to the competitive case with small polluters. The analysis presented here allows a comparison and clarification of the literature on small and large polluters.

In a multi-input framework, with social damages dependent upon total effluents, effluent charges are shown here to result in socially optimal entry, confirming an assertion of Baumol and Oates [2, p. 179, fn. 16], and the important result of Schulze and D'Arge [15]. By taking *average* as well as marginal damages into account, the effluent charge results in optimal firm scale levels and the correct input mix. At the free entry market equilibrium, the firm subject to an effluent tax will operate above or below the private minimum efficient scale depending upon whether average external costs exceed or are less than marginal external costs.

Since public policy to reduce effluents is often directed at reducing the effluent-generating activity, some policy recommendations include a tax on the final output of the polluting firm. In some theoretical analyses, a relation between the output of the firm and the effluent level is identified and a Pigouvian output tax is applied. This approach should not be followed by policymakers since an output tax does not provide firms with the correct incentives for either input substitution or market entry. It is shown that while a lump-sum transfer is needed to provide correct entry incentives when a Pigouvian output tax is applied, no such transfer is needed if the tax is correctly applied to effluents, contrary to [5]. Even if optimal firm scale and

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industry size may be attained with an output tax—lump-sum transfer scheme, firms will not have proper incentives for input substitution. This is significant since output taxes will not lead firms to undertake optimal pollution abatement and effluent treatment.

Effluent standards are reexamined in a free entry framework and are shown to create long-run distortions even if firms are identical. If the optimal per firm standard is employed, excessive entry will occur driving total pollution above the social optimum and driving the production level of each firm below the socially minimum efficient scale. If the policymaker must meet an overall standard of environmental quality using quotas, this will require the share of each firm to be set below the optimal per firm level.

2. THE SOCIAL OPTIMUM

The regulation of an industry which generates pollution is now examined. Firms are assumed to be identical and to behave competitively. Firms purchase inputs x_j at given input prices r_j, $j = 1, \ldots, m$. A newly established firm incurs positive fixed costs F. The firm's inputs are used to produce output q, with production function $q = f(x_1, \ldots, x_m)$, where f is twice differentiable, increasing and concave. For each vector of inputs firms generate an externality e as an unwanted by-product of production, $e = h(x_1, \ldots, x_m)$, where h is convex and differentiable. The pollution generating function h captures the effects of the *scale* of the firm as well as the effects of different *input combinations* on total effluent production. The function h may be *decreasing* in some inputs to represent the employment of inputs for pollution abatement.

External costs of effluent generation are given by the social damage function $D(E)$ where $E = ne$ represents *total effluent*, the effluent produced by each firm, e, times the number of firms n. The damage function, D, is assumed to be differentiable $dD(E)/dE \equiv D'(E) > 0$ with increasing marginal damage and $D(0) = 0$. Total damages can only depend upon the *total effluent* levels. If the spatial distribution of effluent is important, this can easily be handled by redefining effluent levels at different locations as different pollutants.[2] For both air and water pollution, it is the concentration of pollutants within an air shed or water body that is of greatest importance.

The policymaker's problem is to choose industry output, inputs, and the number of firms in a partial equilibrium setting so as to maximize social welfare subject to technological constraints. Let $P(\cdot)$ represent the market inverse demand. Social welfare is defined as total consumer surplus net of private production costs and external environmental damages. The Lagrangian for the policy maker is

$$L = \int_0^{nq} P(s)\,ds - n\sum_{j=1}^{m} r_j x_j - nF - D(ne) + n\xi(f(x) - q) + n\delta(e - h(x)). \tag{1}$$

where ξ and δ are the shadow prices for the production and effluent constraints,

[2] Although a single effluent model is considered here, the results generalize easily to multiple effluent types.

respectively. The first-order necessary conditions include

$$\frac{\partial L}{\partial q}: P(nq) - \xi = 0 \tag{2}$$

$$\frac{\partial L}{\partial e}: - D'(ne) + \delta = 0 \tag{3}$$

$$\frac{\partial L}{\partial x_j}: - r_j + \xi \frac{\partial f(x)}{\partial x_j} - \delta \frac{\partial h(x)}{\partial x_j} = 0 \qquad j = 1, \ldots, m \tag{4}$$

$$\frac{\partial L}{\partial n}: P(nq)q - \sum_{j=1}^{m} r_j x_j - F - D'(ne)e = 0. \tag{5}$$

The optimal allocation and shadow prices $(q^*, x^*, e^*, n^*, \xi^*, \delta^*)$ solve (2)–(5) and $q = f(x)$, $e = h(x)$. Substituting (2) and (3) into (4) yields

$$P(n^*q^*) \frac{\partial f(x^*)}{\partial x_j} = r_j + D'(n^*e^*) \frac{\partial h(x^*)}{\partial x_j}, \qquad j = 1, \ldots, m. \tag{6}$$

Thus, *the marginal revenue product of each input should equal the marginal factor cost to society*. This implies that the firm's *input mix* is as important as its effluent level. This requires the firm to devote the proper amount of resources to inputs which reduce pollution and to choose the correct level of inputs for abatement and pretreatment of effluents. This result indicates why policies aimed at regulating specific pollution-generating inputs or requiring abatement inputs must fail in comparison with direct regulation of effluents.

The first-order condition (5) requires the number of firms to be such that private profit per unit of effluent for each firm equals marginal social damages. This may be interpreted as a zero economic profit condition. The principal optimality conditions are thus (5) and (6).

3. THE EFFLUENT TAX

3a. *Long-Run Optimality*

Consider the long-run impact of an effluent tax $t > 0$. Given the market price P, each firm chooses its output, inputs, and effluent level subject to its production constraints. The Lagrangian for the firm's problem is therefore

$$L = Pq - \sum_{j=1}^{m} r_j x_j - F - te + \lambda(f(x) - q) + \sigma(e - h(x)). \tag{7}$$

The first-order necessary conditions include

$$\frac{\partial L}{\partial q}: P - \lambda = 0 \tag{8}$$

$$\frac{\partial L}{\partial e}: -t + \sigma = 0 \tag{9}$$

$$\frac{\partial L}{\partial x_j}: -r_j + \lambda\frac{\partial f(x)}{\partial x_j} - \sigma\frac{\partial h(x)}{\partial x_j} = 0, \qquad j = 1,\dots,m. \tag{10}$$

The firm sets the shadow price on the effluent constraint equal to the effluent tax, and the shadow price on the production constraint equal to the market price. Thus, (10) implies

$$P\frac{\partial f(x)}{\partial x_j} = r_j + t\frac{\partial h(x)}{\partial x_j}, \qquad j = 1,\dots,m. \tag{11}$$

Note that the effluent tax t affects the firm's marginal rate of technical substitution by adding the marginal cost of an input in effluent production to the input's factor price.

Besides solving the problem of choosing inputs and outputs, firms make entry and exit decisions. Entry occurs until profits are zero given the effluent tax.[3] Thus, at the long-run market equilibrium, $Pq - \Sigma_{j=1}^{m} r_j x_j - F - te = 0$, where $P = P(nq), q = f(x), e = h(x)$, and the input levels solve (11) for all firms.

Now, let the tax t^* equal the shadow price on the effluent constraint in (1), $t^* = \delta^* = D'(n^*e^*)$. Thus, t^* equals *marginal social damages* at the social optimum.

PROPOSITION 1. *Given the effluent tax $t^* = D'(n^*e^*)$, the long-run market equilibrium is socially optimal.*

Correct incentives for entry are obtained from the *total* tax payment $D'(n^*e^*)e^*$. With free entry, factor payments exhaust revenue at the long-run equilibrium, the market price equals $P(n^*q^*)$, and thus condition (5) is satisfied. Also, the effluent tax, t^*, in Eq. (11) causes the firm to *correctly value the productive inputs* by adding their marginal social cost $D'(n^*e^*)\partial h(x^*)/\partial x_j^*$ to the factor price, so that condition (6) is satisfied.

The economic intuition for the result is as follows. All economic inputs that are priced *competitively* at marginal factor cost are paid more than their variable costs when marginal costs exceed average variable costs. This difference is referred to as quasirent, the return to the input producer's fixed inputs. How does this apply to the environment? The society bearing the burden of pollution damage may be seen as both the *owner* of the environment (air, water, etc.) and the *supplier* of environmental services. Environmental services are, in this case, the storage of pollutants provided at the cost of damages to producers and consumers. The difference between the total tax payment and the total social damages may be interpreted as

[3] Starrett and Zeckhauser [17] note that, in a fixed price model with concave production functions, the charges paid by the polluting firm will not affect its decision as to whether or not to produce. This will not hold if nonconvexities are present, see [17]. The reason that entry is affected by effluent charges in our model is that firms face fixed setup costs and the output price is given by a downward sloping demand function so that price is lowered by entry.

the rent accruing to ownership of the scarce factor, the environment. The environmental rent should not be treated differently from rents to other natural resources such as land or mineral resources. The costs of providing the resource services are social costs and are borne directly by consumers and firms. The total of firm payments equals the *sum of social damage costs* and *environmental rent* and provides incentives for optimal entry. Since firms are small, entry of a firm does not significantly alter the equilibrium rent.

The result in Proposition 1 relies on the assumption that all firms are *small*, with negligible effects on marginal social damages resulting from entry of an additional firm. The papers [6, 9, 14, 10, 3] examine *large scale* entry of firms and show that the total tax bill paid by individual firms may be too great.[4] The argument is that since the firm's tax bill (output times marginal damages) exceeds the damages caused *by that firm's entry*, then entry is excessively discouraged in the long run. These results are correct because when there is a change in the marginal social opportunity cost of a factor over the range of usage of that factor by a firm, the marginal increase in rent paid by an entering firm will exceed the social costs of supplying the factor. Thus, with large-scale entry, there is a need for an adjusted emission tax or permits scheme where the total cost to firms equals the area under the marginal social damage curve. This may be achieved, for example, using a Pigouvian tax–lump-sum transfer scheme or using the rental emission permits plan in Collinge and Oates [6], which satisfies both the short-run optimality condition and long-run entry–exit condition when entry of large polluters occurs.

3b. *The Scale of the Firm*

The *long-run* optimality of the effluent tax with small firms is best illustrated in terms of the social cost of producing the output of an individual firm.

$$C(q) = \min_{(x_j)} \left[\sum_{j=1}^{m} r_j x_j + F + t^* h(x) \right] \tag{12}$$

subject to $f(x) = q$, where $t^* = D'(n^* e^*)$. The problem may be solved for inputs as a function of output for a given tax and given input prices, $x_j = \gamma_j(q; t^*, r) = \gamma_j(q)$, $j = 1, \ldots, m$. The effluent tax leads the firm to equate marginal social cost to output price,

$$\begin{aligned} \frac{dC(q)}{dq} &= \sum_{j=1}^{m} \left(r_j + t^* \frac{\partial h(x)}{\partial x_j} \right) \frac{\partial x_j}{\partial q} \\ &= \sum_{j=1}^{m} P \frac{\partial f(x)}{\partial x_j} \frac{\partial x_j}{\partial q} \\ &= P. \end{aligned} \tag{13}$$

Furthermore, from the zero profit entry condition, we see that price also equals

[4] See Collinge and Oates [6, p. 348], Rose-Ackerman [14, p. 514], Gould [10, p. 560], and Dolbear [9, p. 99]. Schulze and D'Arge [15, p. 766, fn. 8] disagree with Dolbear [9] regarding the excess of tax payments over damage costs. The two cases depend upon whether firms are large or small relative to the marginal damages caused by entry of an additional firm.

average social costs for each firm

$$P = \frac{1}{q^*}\left[\sum_{j=1}^{m} r_j x_j^* + F + t^* e^*\right] = \frac{C(q^*)}{q^*}. \tag{14}$$

Thus $dC(q^*)/dq^* = C(q^*)/q^*$ and *average social costs are minimized.*

Combining (13) and (14) and substituting for $t^* = D'(n^*e^*)$ we obtain

$$\left(\frac{\sum_{j=1}^{m} r_j x_j^* + F}{q^*} - \sum_{j=1}^{m} r_j \frac{\partial x_j^*}{\partial q^*}\right) = \left(D'(n^*e^*)\sum_{j=1}^{m} \frac{\partial h(x^*)}{\partial x_j^*}\frac{\partial x_j^*}{\partial q^*} - \frac{D'(n^*e^*)h(x^*)}{q^*}\right). \tag{15}$$

Thus the difference between the firm's average private costs and marginal private costs equals the negative of the difference between the firm's average external costs and marginal external costs. This implies the following result.

PROPOSITION 2. *Given the optimal effluent tax, if the firm's average damages exceed (are less than) marginal damages, the firm will operate above (below) the private minimum efficient scale.*

This result emphasizes the importance of the effluent tax when average and marginal damages are not equal. One must be very careful in interpreting average damages. These refer to the external costs imposed by a firm per unit of final output q. It is because of returns to scale in the firm's generation of pollutants that the firm may have a decreasing, increasing, or u-shaped average damage curve. This should not be confused with the form of the social damage function $D(ne)$ which is a function of effluents *not outputs*. The firm takes marginal social damages t^* as given. The result explains why the separate entry regulations suggested by Carlton and Loury [5] are not needed when effluents rather than outputs are taxed, see Section 5 below. Since small firms have a negligible impact on marginal social damages, the firm takes marginal social damages t^* as constant. Thus the shape of the firm's external cost function $t^*e(q)$ is strictly dependent upon its own pollution-generating technology.

It should be added that the effluent tax achieves the socially optimal input and output levels without a specification of the effluent-output relation for the firm. In addition, the tax gives incentives for complex input substitution by the firm. There is a tendency on the part of regulators to focus on output controls, input restrictions (e.g., low-sulfur fuel oil) or technological requirements (e.g., the best practicable BPT or available BAT technology requirements in water quality regulation). The effluent tax avoids these thorny regulatory requirements.

4. TRADEABLE EFFLUENT PERMITS

The literature on effluent permits has focused attention upon industries with a fixed number of firms.[5] The optimality properties of permits carry over to the free

[5]See the important work of Dales [7, 8] and the survey in Tietenberg [18]. The proper functioning of the permit market requires a sufficient number of firms, see Starrett and Zeckhauser [17] on "thinness" of markets. Hahn [11] presents a simple model of firm demand for effluent permits where the firm selects its effluent level based upon the cost of abatement opportunities. The demand for permits is examined here under more general assumptions on firm technology.

entry case. Suppose that E permits are issued where l indicates the number of permits purchased by a firm. The market price of a permit is v. The firm's supply function as well as its demand for inputs will depend upon the price of permits. We can also obtain the firm's demand for permits. The firm solves a Lagrangian problem analogous to (7) where $l = e$, $v = t$, and $\beta = \sigma$ is the shadow price on the pollution constraint,

$$L = Pq - \sum_{j=1}^{m} r_j x_j - F - vl + \lambda(f(x) - q) + \beta(l - h(x)). \tag{16}$$

The constraint that permits exceed effluents, $l \geqslant h(x)$ is always binding since the firm will never purchase excess permits. The firm sets the shadow price on the permit constraint $l \geqslant h(x)$ equal to the market price of a permit, $\beta = v$. The firm's problem (16) may be solved for the firm's supply function, input demand, and permit demand functions, $q^s = q(P, r, v)$, $x_j^D = x_j(P, r, v)$, $j = 1, \ldots, m$, and $l^D = l(P, r, v)$.[6]

Since entry depends on the price of permits, the aggregate market demand for permits is $L^d(v) = n(v)l^D(v)$. This is downward sloping in v.[7] The demand for permits has an interesting economic interpretation. It is the marginal valuation of environmental services to the polluting industry. Given the total number of permits issued $\overline{E}$, the market price v^* allocates the permits across firms in the industry (see Fig. 1a). If the total number of permits issued equals the socially optimal pollution level E^*, then the optimum level is attained by the market in permits (see Fig. 1b). At v^*, the marginal value of environmental services equals the marginal social cost of providing those services represented by the marginal damage function. If the area under the marginal damage curve equals total damage costs then the shaded area in Fig. 1b is the rent accruing to the owners of the environmental resource.

Given the total number of effluent permits equal to the social optimum E^*, compare the solution to the firm's problem (16) with the social optimum, (2) to (5).

PROPOSITION 3. *Given the total number of permits E^*, then, at the equilibrium in the market for effluent permits, the optimal assignment of effluent levels is achieved and the long-run industry equilibrium is optimal.*

To see this, let $t = v^*$ in (8) to (10). Suppressing P and r, the firm's inputs $x^D = x(v^*)$ and output $q^s = q(v^*)$ solve (11) and since $l = h(x)$ is a binding constraint, $n(v^*)l^D(v^*) = E^*$. Since the zero profit entry condition holds at the market equilibrium $v^* = D'(E^*)$, and the optimality conditions (5) and (6) are satisfied.

[6] Sufficient conditions for the permit demand to be downward sloping are given in Spulber [16].

[7] To see this, write the zero profit entry condition with permits, $\pi(r, v, n) = 0$, where $\pi(r, v, n) = P(nq)q - \sum_{j=1}^{m} r_j x_j - F - vl$. Differentiate π with respect to v holding n constant, $\partial\pi/\partial v = -l^D(v)$. Differentiate π with respect to n,

$$\frac{\partial \pi}{\partial n} = \frac{-P'(nq)C''(q)q}{P'(nq)n - C''(q)} < 0$$

where $C(q; R, v)$ is the firm's cost function. Thus, the equilibrium number of firms is decreasing in the permit price, $\partial n/\partial v = -(\partial\pi/\partial v)/(\partial\pi/\partial n) < 0$. Therefore the aggregate demand is downward sloping since $\partial L^D(v)/\partial v = n(v)\partial l^D(v)/\partial v + l^D(v)\partial n(v)/\partial v$.

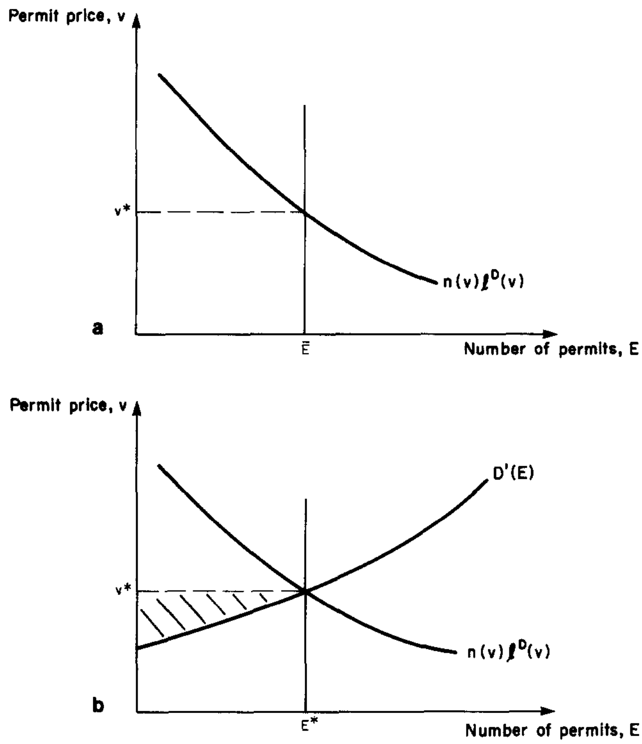

FIGURE 1

An additional interpretation of the permit market equilibrium is obtained by rewriting (11),

$$\frac{P(n(v^*)q^s(v^*))\,\partial f(x^D(v^*))/\partial x_j - r_j}{\partial h(x^D(v^*))/\partial x_j} = v^*, \qquad j = 1,\ldots,m. \tag{17}$$

Thus, the net marginal revenue from the environmental services associated with each factor equals the price of the environmental service, that is, the permit price. The net returns to use of the environmental service may be interpreted as an inverse demand or willingness to pay schedule.

5. THE OUTPUT TAX

A Pigouvian tax is generally taken to mean a tax (or subsidy) per unit of emissions generated by a firm.[8] Frequently, for convenience or as a teaching device, the Pigouvian tax is described as a tax on the final output of the firm and the output is

[8] Baumol and Oates [2, p. 45] state that "[i]nputs and outputs that generate smoke are, of course, subject to tax but only in proportion to the smoke they produce." In practice, there may not be a proportional relationship between a firm's final output and its emissions. As Plott [13] finds, a tax per unit of output may increase a firm's emissions if they are an inferior input, see also Baumol [1, p. 311, fn. 6]. Burrows [3, fn. 2] notes that the original Pigou analysis refers to "'taxes' on external-cost-generating activities such as petrol duty and motor vehicle license tax." Burrows [3, p. 495] interprets the Pigouvian tax as being a charge per unit of effluent emission.

in some way associated with the firm's effluent emissions. In this context, the tax creates incentives for the firm to reduce its output and in so doing reduce its emissions. This approach sometimes causes confusion since it seems to imply that public policy to reduce emissions can be achieved with output taxes. Further confusion exists in the theoretical regulation literature where conclusions about Pigouvian taxes are obtained from models with output taxes. This section shows why an output tax cannot work when input substitution affects the effluents generated by the firm. Even if output can be directly associated with effluents, the output tax cannot work since it will result in nonoptimal entry of firms into the regulated industry. Insight is gained into how the effluent tax provides correct entry incentives by examining the transfer needed to correct the output tax.

Consider first the short run with a fixed number of firms n. Assume that only a single input is used in production, $x \in R_+$. Then, $e = h(x) = h(f^{-1}(q))$. Thus, the marginal damages per firm may be calculated as a function of output

$$\frac{1}{n}\frac{\partial D\big(nh\big(f^{-1}(q)\big)\big)}{\partial q} = D'(ne)\frac{h'(x)}{f'(x)}. \tag{18}$$

The Pigouvian output tax, τ, equals marginal damages per firm, with e^*, x^* as the optimal effluent and input levels, $\tau \equiv D'(ne^*)h'(x^*)/f'(x^*)$. *Given the Pigouvian output tax τ and given one input, the optimal allocation is achieved.* The firm chooses its input x to maximize profit, $\pi = Pf(x) - \tau f(x) - rx$. The first-order condition for the firm is then $(P - \tau)f'(x^*) = r$. Substituting for τ and rearranging terms, the optimal effluent equation for a fixed number of firms (6) is obtained,

$$Pf'(x^*) = r + D'(ne^*)h'(x^*) \tag{19}$$

for $e^* = h(x^*)$. The result that the output tax is optimal holds because only the *scale* of each firm is important. The result will not hold if the firm may substitute inputs in production, each of which may have different marginal products in output and pollution production.

For $x \in R_+^m$, $m \geqslant 2$, and a constant number of firms, the output tax does not affect the firm's private cost minimization. Thus, for an output tax τ

$$(P - \tau)\frac{\partial f(x)}{\partial x_j} = r_j, \qquad j = 1, \ldots, m, \tag{20}$$

so that the firm's input choices are not directly affected,

$$\frac{\partial f(x)/\partial x_j}{\partial f(x)/\partial x_i} = \frac{r_j}{r_i}, \qquad i \neq j; \qquad i, j = 1, \ldots, m. \tag{21}$$

This differs from the social cost minimization condition, from (11),

$$\frac{\partial f(x)/\partial x_j}{\partial f(x)/\partial x_i} = \frac{r_j + D'(nh(x))\,\partial h(x)/\partial x_j}{r_i + D'(nh(x))\,\partial h(x)/\partial x_i}. \tag{22}$$

Therefore, under an output tax the firm will have an incorrect input mix and the firm may not engage in the right amount of effluent pretreatment activities.

PROPOSITION 4. *For $m \geqslant 2$ and a constant number of firms, the output tax does not achieve an optimal allocation.*

The Pigouvian output tax does not yield an optimal outcome in the long run even for the single input case. Given output tax τ equal to $D'(ne)h'/f'$ the entry equation is

$$P(nq)f(x) - D'(ne)\frac{h'(x)}{f'(x)}f(x) - rx - F = 0. \tag{23}$$

For the effluent tax equal to $D'(n^*e^*)$, the entry equation is

$$P(n^*q^*)f(x^*) - D'(n^*e^*)e^* - rx^* - F = 0. \tag{24}$$

Clearly, a lump-sum tax (or subsidy) per firm equal to

$$T^* = f(x^*)\left[D'(n^*e^*)\frac{e^*}{f(x^*)} - D'(n^*e^*)\frac{h'(x^*)}{f'(x^*)}\right] \tag{25}$$

will result in the correct number of firms entering the industry. This follows from the optimality of the Pigouvian output tax when the proper entry incentives are provided. This implies the following result.

PROPOSITION 5. *For a single input and with free entry of firms, the optimal effluent tax equates the difference between the firms' marginal private costs and average private costs to the transfer needed to correct the Pigouvian tax per unit of output.*

To see this, note that from Proposition 2, the difference between marginal private costs (MPC) and average private costs (APC) equals

$$\text{MPC} - \text{APC} = \left[D'(n^*e^*)\frac{e^*}{q^*} - D'(n^*e^*)\frac{h'(x^*)}{f'(x^*)}\right] \tag{26}$$

when the optimal effluent tax is applied. But this difference exactly equals T^*/q^* as given by Eq. (25).

Consider now the regulation literature in which output taxes are used. Schulze and D'Arge [15] show that an output tax leads to optimal resource allocation with small firms given the damage function $D(nq)$. In our framework, this is true if for a single input $T^* = 0$ in (25), or $h(x)/f(x) = h'(x)/f'(x)$ which holds, for example, when effluents are linear in output, $e = h(x) = b \cdot f(x)$. In this case, the result in [15] supports our model.

[9]Clearly an effluent tax may fail to be optimal if the damage function has the form $D(n, e)$. However, this form is meaningless in general since only total effluents can cause damage, not the number of firms.

Carlton and Loury [5] assert that the Pigouvian effluent tax does not lead to long-run optimality without a corrective lump-sum transfer. Carlton and Loury [5] employ a social damage function dependent upon the scale and number of firms $\bar{D}(n, q)$ and study the Pigouvian output tax.[9] It is shown in [5] that the lump-sum transfer equals

$$T = q\left[\frac{1}{q}\frac{\partial \bar{D}(n,q)}{\partial n} - \frac{1}{n}\frac{\partial \bar{D}(n,q)}{\partial q}\right]. \tag{26}$$

and that (p. 564): "The optimal lump sum tax... will be positive (negative) when average pollution damage is falling (rising) at the optimal firm output."

In [5], the failure of the Pigouvian tax is attributed to the difficulty of trying to control the number of firms and the scale of firms with just one policy instrument. Proposition 5 above suggests that the problem lies not in the *number* of instruments. Rather, the problem lies in the *choice* of instruments. When a tax is placed on the firm's effluent, the firm's average and marginal private costs will differ by exactly the amount of the transfer per unit of output that is needed to correct the Pigouvian output tax. The output tax–transfer scheme in [5] generally will not work in a multi-input model. First, to verify the one input case within our framework, the damage function $\bar{D}(n, q)$ is obtained by substituting for $e = h(x)$, $x = f^{-1}(q)$, $\bar{D}(n, q) \equiv D(nh(f^{-1}(q))) = D(ne)$. Thus, we have

$$\frac{\partial \bar{D}(n,q)}{\partial n} = D'(ne)e \tag{27}$$

$$\frac{\bar{D}(n,q)}{\partial q} = D'(ne)\frac{nh'(x)}{f'(x)} \tag{28}$$

where $x = f^{-1}(q)$. Substituting for $\partial \bar{D}(n,q)/\partial n$ and $\partial \bar{D}(n,q)/\partial q$ in (26) clearly $T = T^*$ as obtained in our optimality condition (25). The optimality of a Pigouvian output tax and lump-sum transfer system in a one-input framework should not be used as a guide to policy in a multi-input framework. The Carlton and Loury [5] result depends heavily on the assumption that pollution damages are a function of the *scale* and *number* of firms. As we have shown above, the firm's input mix is of importance. It is generally not possible to associate firm scale with effluent and thus with social damages. Thus, the Pigouvian output tax–lump-sum transfer scheme in [5] may be adjusted to obtain any desired scale and number of firms but the firm's input mix and the resulting effluent levels will be far from optimal. Equation (20) may be solved for $x_j = x_j(q)$, $j = 1, \dots, n$. Then the social damage function may be written as a function of n and q, $\bar{D}(n, q) \equiv D(ne) = D(nh(x_1(q), \dots, x_n(q)))$. The Pigouvian output tax and lump-sum transfer may be adjusted to set q and n. However, the social costs of achieving this allocation will be far greater than necessary due to the misallocation of inputs represented by (20). The misallocation will not occur if h is a positive linear transformation of f. Given the different effects of various inputs such as labor, capital, resources, and energy on pollution generation and abatement, this seems highly unlikely in practice.

The negative result of Burrows [3] assumes that damages are *additive* across firms, i.e., total damages equal $nD(q)$, see [3, fn. 5]. Thus external damages caused by firms are *local* and *independent*. An alternative specification of different pollution impacts at N locations might allow entry of firms at each location. Let the regulator choose industry size n_i and output q_i at each location $i = 1, \ldots, N$ to maximize

$$\int_0^Q P(s)\,ds - \sum_{i=1}^{N} n_i C(q_i) - \sum_{i=1}^{N} n_i F - \sum_{i=1}^{N} D_i(n_i q_i),$$

where $Q = \sum_{i=1}^{N} n_i q_i$ is total output and $D_i(n_i q_i)$ is damage at each location. An output tax at each location $t_i = D_i'(n_i q_i)$ will lead to correct output *and* entry incentives, as in [15].

6. THE EFFLUENT STANDARD

It is well known that for a fixed number of firms, effluent standards may not yield an optimal allocation when firm technologies differ.[10] When firms are identical, standards are generally viewed as the quantity dual of the optimal effluent tax. However, effluent standards fail in the long run even when firms are identical. The economic intuition is that effluent standards give firms a valuable property right to a particular use of the environment thus encouraging excessive entry.

Suppose that a regulator wishes to select per-firm emissions levels using an effluent standard. For simplicity assume that there are two productive inputs used in production, $q = f(x_1, x_2)$ and that pollution is generated by use of one of the inputs $e = h(x_2)$. The optimal per-firm pollution level e^* solves (7). Let $\bar{x}_1, \bar{x}_2, \bar{q}, \bar{n}$ denote the market equilibrium in the long run given the effluent standard e^*. Note that since the constraint is still binding, $\bar{e} = e^*$ and $\bar{x}_2 = x_2^*$. The market equilibrium is defined by the marginal revenue product condition, $P(\bar{n}\bar{q}) f_1(\bar{x}_1, \bar{x}_2) = r_1$, and the zero profit entry condition, $P(\bar{n}\bar{q}) f(\bar{x}_1, \bar{x}_2) - r_1\bar{x}_1 - r_2\bar{x}_2 - F = 0$. These conditions imply the following.

PROPOSITION 6. *Given an across-the-board effluent standard equal to the socially optimal per-firm pollution level, each firm operates below the social minimum efficient scale, excessive entry occurs, and total pollution exceeds the social optimum.*

The argument is as follows. The entry condition implies

$$P(n^*q^*) f(x_1^*, x_2^*) - r_1 x_1^* - D'(n^*e^*)e^* = P(\bar{n}\bar{q}) f(\bar{x}_1, \bar{x}_2) - r_1\bar{x}_1. \quad (29)$$

Thus,

$$P(n^*q^*) f(x_1^*, x_2^*) - r_1 x_1^* > P(\bar{n}\bar{q}) f(\bar{x}_1, \bar{x}_2) - r_1\bar{x}_1. \quad (30)$$

From the marginal revenue product condition, (30) implies

$$f^*(x_1^*, x_2^*)/f_1^*(x_1^*, x_2^*) - x_1^* > f(\bar{x}_1, \bar{x}_2)/f_1(\bar{x}_1, \bar{x}_2) - \bar{x}_1. \quad (31)$$

Thus, $x_1^* > \bar{x}_1$ since $\bar{x}_2 = x_2^*$ and $(f/f_1) - x_1$ is increasing in x_1. Given $x_1^* > \bar{x}_1$, output is therefore greater at the social optimum $q^* > \bar{q}$ and $f_1(x_1^*, x_2^*) < f_1(\bar{x}_1, \bar{x}_2)$. Thus, from the marginal revenue product condition $P(n^*q^*) > P(\bar{n}\bar{q})$ which implies $n^* < \bar{n}$ because demand is downward sloping.

[10] See Baumol and Oates [2], Kneese and Bower [12, p. 132] and Dales [8].

The effluent standard does not affect total profits which therefore allows *excessive entry*. This raises total output, thus lowering the market price for any output level and leading to lower output for each firm. The increased entry leads to greater total pollution. This last result confirms a conjecture of Burrows [4, fn. p. 362], that "a larger industry under regulation could, however lead to higher *industry* use of the environment." With free entry, lower prices cause firms to reduce the unregulated inputs leading to lower output per firm. This may create a confusing situation for a regulator who will face complaints from firms in the industry who must cut back their production at the same time that more firms are entering the industry. The firms may call for entry restrictions as a means of controlling pollution. Thus, regulators may be forced to a scheme that combines per-firm emissions requirements with limited entry regulation.

Suppose that the regulator is attempting to meet an *overall legal environmental standard* E by assigning per-firm shares e such that $ne \leqslant E$. The Lagrangian for the policymaker's problem is then

$$L = \int_0^{nq} P(s)\,ds - n\sum_{j=1}^{m} r_j x_j - nF + n\xi(f(x) - q) + \alpha(E - nh(x)) \quad (32)$$

where the regulator chooses x, q, and n, where $\alpha \geqslant 0$ is a Kuhn–Tucker multiplier. The problem (32) yields an identical solution to the social optimum problem (1) if $E = n^*e^*$. Let $(\bar{q}, \bar{x}, \bar{e}, \bar{n}, \bar{\xi}, \bar{\alpha})$ solve (32). An effluent tax equal to the shadow price on the overall effluent constraint $\bar{\alpha}$ will yield an optimal allocation in the long run. Thus, the effluent tax is the social welfare maximizing policy instrument for meeting an overall environment standard. For a binding environmental constraint, the shadow price $\bar{\alpha}$ is positive so that an across-the-board standard assigning equal shares of the environmental constraint to each firm equal to $E/\bar{n}$ will result in excessive entry thus raising the social costs of achieving the environmental standard.

PROPOSITION 7. *Given a binding overall environmental standard and free entry, the across-the-board share per firm must be set below the optimal level* $\bar{e}$.

Proposition 7 follows from Eq. (4) which implies that firm profits are positive at the social optimum when $\bar{\alpha} > 0$,

$$P(\bar{n}\bar{q})\bar{q} - \sum_{j=1}^{m} r_j\bar{x}_j - F = \bar{\alpha}h(\bar{x}) > 0. \quad (33)$$

Thus, meeting a legal standard by an across-the-board quota will place excessive burdens upon firms. If fixed costs are relatively small relative to net revenues, excessive entry will require tight pollution restrictions on individual firms. Regulators will face requests from firms to limit entry or to approach the lower standard $\bar{e}$ gradually perhaps by placing stricter requirements upon newly entering firms.

7. CONCLUSION

The results obtained in this paper reaffirm the importance of directing public policy toward pricing the use of the environment. The policy instruments which achieve long-run optimality in a competitive market with small firms are the effluent tax and the tradeable effluent permit. Direct intervention through output taxes or output controls, entry tariffs, or restrictions, or effluent constraints will create further distortions in the allocation of resources. Given an effluent charge or permit price equal to marginal social damages, the total of firm payments equals the sum of environmental rent and external damages. These two components provide optimal incentives for entry in the long run.

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