Student's Name: id#:

Exponential, X≥0

f(x)=ße$^{-ßx}$

$$\phi(t) = \frac{\theta}{\theta - t}$$

where the mean is ß$^{-1}$ and the variance is ß$^{-2}$ P(X>c)=exp(-Xc)

Bernoulli, X=0,1

$$P(X = x) = p^x(1-p)^{1-x} \qquad \phi(t) = 1 - p + pe^t$$

With a mean of p and variance of p(1-p)

Binomial, X=0,1,2,3,....,n

$$P(X = x) = \binom{n}{x} p^x(1-p)^{n-x} \qquad \phi(t) = (1 - p + pe^t)^n$$

With a mean of np and variance of np(1-p)

Poisson, X=0,1,2,3,....

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad \phi(t) = e^{u(e^t - 1)}$$

with a mean of λ and variance of λ

Normal, $-\infty < X < \infty$

$$f(X) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(X-\mu)^2\right) \qquad \phi(t) = \exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$$

with a mean of μ and variance of σ^2

You are permitted one double sided page of notes. No electronic devices are permitted or needed.

Show that if X_1 , X_2,...X_n are iid bernoulli trials with parameter p, then $Z=\sum_i X_i$ is a binomial rv with parameters n and p. (2pts)

A Manuscript is sent to a typing firm consisting of typists A, B and C. If it is typed by A, then the number of errors made is a Poisson random variable with mean 1, if typed by B then the number of errors is a Poisson random variable with mean 3 and if typed by C then the number of errors is a Poisson random variable with mean 5. Let X denote the number of errors in the typed manuscript. Assume that typist B does the work with probability 1/2 and typists B and C do the work with probability 1/4 each.

Find E(X) (3pts)

Find Var(X) (3pts)

A fair coin is flipped.

a) What is the probability that the first 4 coin tosses turn up HHHH? (1pt)

b) If you keep tossing coins until the pattern HHHH turns up, how long would you expect to wait? (2pts)

Consider the sequence of interest HTHTT. Define state 0 as the state where none of the tosses from the sequence have occurred. Define State 1 as H, state 2 as HT, state 3 as HTH,..., state 5 as HTHTT. Produce the probability transition matrix with all the probabilities for all of these states and determine the expected time until the sequence has occurred. (4 pts)

X is an exponential random variable. For some constant M, find the distribution of (X | X>M) (3pts)

(3pts) For some continuous random variable Y with unknown distribution where Y≥0 show that:

$$P(X \geq a) \leq \frac{E(X)}{a}$$

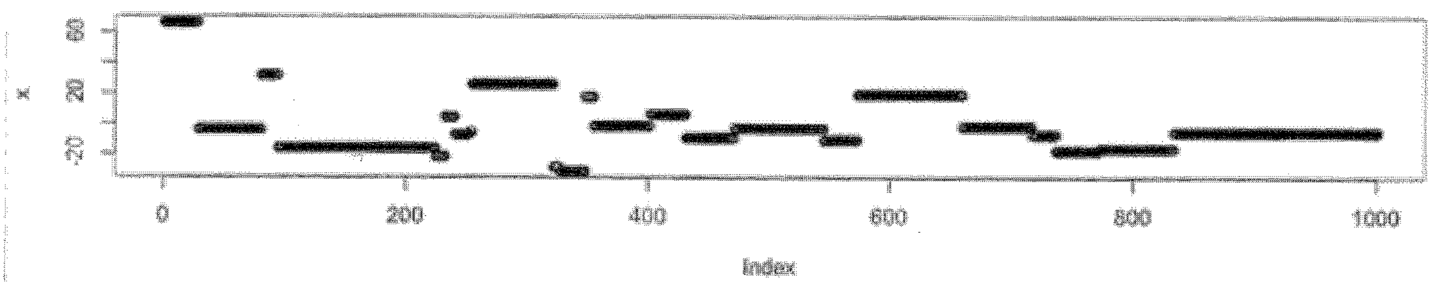
To help cars see pedestrians Salt Lake City has installed buckets on either side of a crosswalk and has distributed 4 flags into those buckets. If a flag is available when a pedestrian arrives, the pedestrian will take a flag from the bucket and will carry it across the street with them and place it in the bucket. Flags therefore transfer from one side of the street to the other. If no flags are available an individual will cross the street without a flag in hand. Individuals are equally likely to begin on either side of the street.

What is the long term proportion of street crossings which will occur without a flag in hand? (8pts)

Along with your answer, provide the probability transition matrix for this stochastic process.

You run a Metropolis Hastings Markov chain attempting to produce a sample from a continuous distribution. After 1000 sampled values you plot the accepted values of X_t vs iteration (index t).

What does this plot tell you? If there is a problem how do you fix it? (4pts)



There are 3 probability transition matrices for Markov chains given below. All states should be labelled starting at 0,1,...

Find the classes of the Markov chains. (3pts)

Label each class, state or Markov chain (which ever is most appropriate) with as many of these labels as possible: Transient (T), Recurrent (R), Irreducible (I) (3pts)

$$P_1 = \begin{Vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 & 0 \end{Vmatrix}$$

$$P_2 = \begin{Vmatrix} 1/2 & 0 & 1/2 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \end{Vmatrix}$$

$$P_3 = \begin{Vmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{Vmatrix}$$

8.At a gas station customers arrive according to a constant Poisson process with rate 6 per hour and the shop has room for at most 2 customers. Customers will not line up when the 2 spots are full. The service time is exponentially distributed with a mean of 5 minutes for each customer in the gas station. What fraction of the time is the gas station empty? (5pts)

A personal chef cooks meals for Gary every day that are either soup (s), toast (t) or rice (r). Let p_i be the probability that Gary enjoys meal i. and suppose that p_s=.3, p_t= .6 and probability p_r=.9. If Gary enjoys the meal, then the next meal is equally likely to be any of the three types. If Gary does not enjoy the meal then the chef will always make soup. What proportion of the meals are type i, where i=s,t,r? (5pts)

Bonus question (1point):

A bonus assignment was offered during this course. What topic/application did the bonus assignment focus on?